INTRODUCTION TO DYNAMICAL SYSTEMS AND CHAOTIC SYSTEMS

EXERCISE 4

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1. Speed Up the Basics

For the following linear systems, plot the phase portrait and classify all fixed points.

(1) $\dot{x} = y$, $\dot{y} = -2x - 3y$,
(2) $\dot{x} = 5x + 10y$, $\dot{y} = -x - y$,
(3) $\dot{x} = 3x - 4y$, $\dot{y} = x - y$,
(4) $\dot{x} = 5x + 2y$, $\dot{y} = -17x - 5y$,
(5) $\dot{x} = 4x - 3y$, $\dot{y} = 8x - 6y$,
(6) $\dot{x} = y$, $\dot{y} = -x - 2y$.

2. Damped Harmonic Oscillator

The damped harmonic oscillator is given by the equation

$$m\ddot{x} + b\dot{x} + kx = 0$$

where $m > 0$ denotes the mass of the particle, $b > 0$ is a friction constant, and $k > 0$ describes the strength of the potential field.

(1) Rewrite the equation as a linear two-dimensional system. You may also want to get rid of one of the constants, preferably m.
(2) What type of fixed point is the origin depending on the parameters? Sketch the phase portrait in each case.
(3) Find a one-dimensional system that arguably describes well the case with high friction.

3. Phase Portraits

For the following non-linear systems, sketch the vector field and the phase portrait. In particular, what are the fixed points and what is their type?
Hint: You may want to conform your sketched portraits using the computer.

(1) $\dot{x} = x - y$, $\dot{y} = 1 - e^x$,
(2) $\dot{x} = x - x^3$, $\dot{y} = -y$,
(3) $\dot{x} = x(x - y)$, $\dot{y} = y(2x - y)$,
(4) $\dot{x} = x^2 - y$, $\dot{y} = x - y$.

What are stable and unstable manifolds in each case?

4. Working Backwards

For each of the following cases, find and sketch a phase portrait that satisfies the given property

(1) These are precisely two saddle fixed points and a single trajectory that connects them (heteroclinic).
(2) There are precisely two saddle fixed points and not a single trajectory connects them (no heteroclinic).

Optional: Find (simple) equations for the phase portraits.

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5. Linearization is not Enough, Example 1

Consider the system $\dot{x} = xy$, $\dot{y} = x^2 - y$.

(1) Show that the origin is the only fixed point.
(2) What type of fixed point is the origin? Sketch the vector field around the origin.

6. Linearization is not Enough, Example 2

Consider the system $\dot{x} = -x^3 - y$, $\dot{y} = x$.

(1) Show that the origin is the only fixed point.
(2) What type of fixed point is the origin? Sketch the vector field around the origin.