Introduction to Continuous Logic
Department of Mathematics and Statistics, University of Helsinki
Fall 2015
Exercise 6

1. For φ an $L(A)$-formula, let $\tilde{\varphi}$ be the function $\tilde{\varphi} : S_n(T_A) \to [0, 1]$ defined on p. 45 in the material. Show that the logic topology is the coarsest topology for which all the functions $\tilde{\varphi}$ are continuous.

2. Show that the type space is compact under the logic topology but not necessarily under the d-metric.

3. Show that the d-topology is finer than the logic topology on $S_n(T_A)$. When are the topologies equivalent?

4. (Topological Tarski-Vaught Test). Let $\mathcal{M}$ be a structure, and let $A \subseteq M$ be a closed set. Show that the following are equivalent:

(1) The set A is (the domain of) an elementary substructure of $\mathcal{M}$.
(2) The set of realized types $\{\mathrm{tp}_{\mathcal{M}}(a/A) : a \in A\}$ is dense in $S_1(A)$.

5. If $F \subseteq S_n(T)$ is closed in the logic topology and $\varepsilon > 0$, define the closed ε-neighborhood of F:

$$F^{\varepsilon} = \{p \in S_n(T) : d(p, F) \leq \varepsilon\}.$$

Show that F^{ε} is closed in the logic topology.