Introduction to Continuous Logic
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Exercise 5

1. Assume $\mathcal{M}_0 \subseteq \mathcal{M}_1 \subseteq \mathcal{M}_2$, $\mathcal{M}_0 \preccurlyeq \mathcal{M}_2$ and $\mathcal{M}_1 \preccurlyeq \mathcal{M}_2$. Show that $\mathcal{M}_0 \preccurlyeq \mathcal{M}_1$.

2. Show that if $\mathcal{M}$ is κ-saturated, then density$(\mathcal{M}) \geq \kappa$ or $\mathcal{M}$ is compact.

3. A model $\mathcal{M}$ is *saturated*, if it is density$(\mathcal{M})$-saturated. Show that if CH holds (i.e., $2^{\aleph_0} = \aleph_1$) then any separable model of a countable vocabulary L can be extended to a saturated model of density $\aleph_1$.

4.
(1) Assume L is countable and $\mathcal{M}$ is an L-structure, $\kappa \geq \aleph_0$, $A \subset M$, $|A| < \kappa$. Denote by $\mathcal{M}_A$ the $L(A)$-structure one gets by giving the new constant symbols their natural interpretation. Show that if $\mathcal{M}$ is κ-saturated, then so is $\mathcal{M}_A$. (Note: in continuous model theory there is also a notion of *approximate saturation* for which the claim is not true.)
(2) Show that $\mathcal{M}$ is κ-saturated if and only if it realizes all types in $S_1(T_A)$ for $A \subset M$ with $|A| < \kappa$.

5. Prove remark 8.2 of the material.