# HARMONIC ANALYSIS AND SQUARE FUNCTIONS: HINTS FOR THE EXERCISE SET 2

(5) Recall that sets can be approximated in measure by open sets (given $\epsilon > 0$ there exists open set $U \supset A$ so that $\mu(U) \leq \mu(A) + \epsilon$), and choose appropriate maximal dyadic cubes.

(6) Decompose the summation over the dyadic cubes as follows

$$\sum_{j \in \mathbb{Z}} \sum_{\substack{Q \in \mathcal{D}_0 \\ 2^j < |\langle f \rangle_Q^\mu| \leq 2^{j+1}}},$$

for each fixed $j$ consider appropriate maximal dyadic cubes, and try to use the assumption

$$\frac{1}{\mu(R)} \int_R \Big[ \sum_{\substack{Q \in \mathcal{D}_0 \\ Q \subset R}} |A_Q(x)|^2 \Big]^{p/2} d\mu(x) \leq [\mathrm{Car}_p((A_Q)_{Q \in \mathcal{D}_0})]^p \mu(R)$$

with $R$ being one of the chosen maximal dyadic cubes. Recall also that the dyadic maximal operator is $L^p(\mu)$ bounded.

(7) Fix $P_0 \in \mathcal{D}_0$, where you want to show the claim. Choose maximal cubes $R \in \mathcal{D}_0$ so that $R \subset P_0$ and

$$\Big| \sum_{\substack{Q \in \mathcal{D}_0 \\ R \subsetneq Q \subset P_0}} \varphi_Q(x) \Big| > 1, \qquad x \in R.$$

The left-hand side is constant on $R$ so this makes sense. Denote these cubes $\mathcal{R}_1$ and set $S_1 = \bigcup_{R \in \mathcal{R}_1} R$. What properties do you have? Keep iterating this construction.