Application of Markov chains to market analysis

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Consider a market of four telecommunications service providers. Denote by horizontal vector $x = [x_1, x_2, x_3, x_4]$ the market shares of the four companies:

$$\begin{aligned} x_1 &= \text{Market share of TNT,} \\ x_2 &= \text{Market share of Tomera,} \\ x_3 &= \text{Market share of Kaviolinja,} \\ x_4 &= \text{Market share of Oranki.} \end{aligned}$$

Every element of x satisfies $0 \leq x_j \leq 1$, because each company has a nonnegative percentage of the market. Additionally we have $x_1 + x_2 + x_3 + x_4 = 1$, since we assume that these four companies are the only choices for customers.

Assume that on each day the following happens:

- Each customer of TNT moves to Tomera with probability $\frac{1}{2}$, to Kaviolinja with probability $\frac{1}{3}$, and to Oranki with probability $\frac{1}{6}$.
- Each customer of Tomera moves to TNT with probability 0, to Kaviolinja with probability $\frac{1}{4}$ and to Oranki with probability $\frac{1}{4}$. Each customer will stay with Tomera with probability $\frac{1}{2}$.
- Each customer of Kaviolinja moves to TNT with probability $\frac{1}{4}$, to Tomera with probability $\frac{1}{4}$, and to Oranki with probability $\frac{1}{4}$. Each customer will stay with Kaviolinja with probability $\frac{1}{4}$.
- Each customer of Oranki moves to TNT with probability $\frac{1}{10}$ and stays with Oranki with probability $\frac{9}{10}$.

Let us define probabilities p_{ij} as follows:

$$p_{ij} = \text{probability to switch from operator } i \text{ to operator } j.$$

Use Matlab to construct the matrix $P = [p_{ij}]$. Check that the sum of the elements in each row is one (use command `P'` to traspose P and the command `sum` to calculate the sum). This is an example of a *stochastic matrix*.

Assume that the market shares are initially $x^{(0)} = [\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}]$. This means that all the four companies have the same number of customers. Then we can compute the next day's market shares using the formula

$$x^{(1)} = x^{(0)} P.$$

(Why?) Furthermore, the market shares on the following day are given by the vector $x^{(2)} = x^{(1)} P = x^{(0)} PP$, and after n days by the vector $x^{(n)} = \cdots = x^{(0)} P^n$. Use Matlab to calculate a few of the first members in the sequence

$$x^{(0)}, x^{(1)}, x^{(2)}, x^{(3)}, \ldots$$

What do you observe?