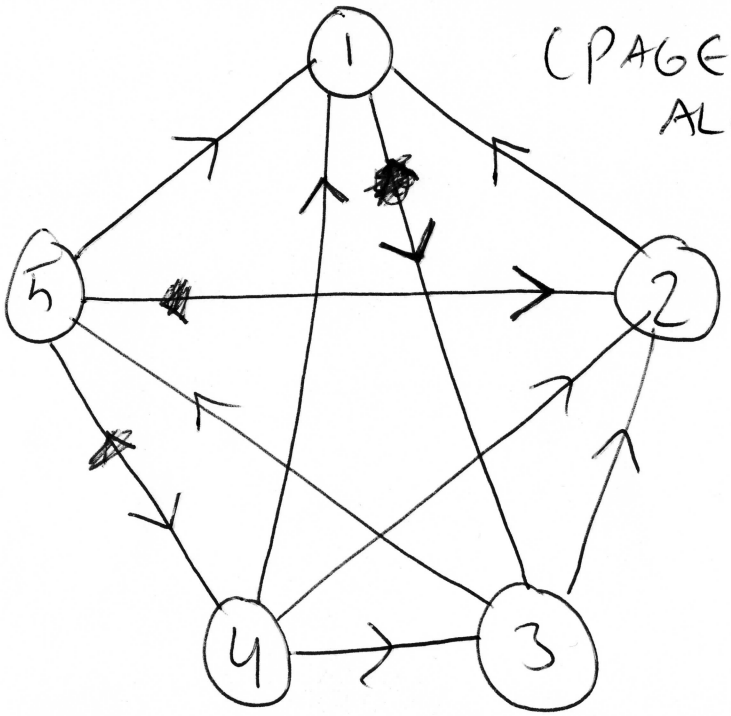
FIVE-PAGE INTERNET EXAMPLE (PAGERANK ALGORITHM)



$a_{ij} = 1$ if page j has a link to page i, and $a_{ij} = 0$ otherwise.

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

associate ranking r_i to page i in such a way that $r_i > r_j$ indicates that page i is more highly ranked than page j.

Let's require that

$$0 \leq r_i \leq 1 \quad \text{and}$$

$$r_1 + r_2 + \cdots + r_5 = 1 .$$

Define ranking vector

$$\vec{r} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{bmatrix} .$$

Furthermore, let's insist that the ranking r_i of page i should be proportional to the sum of the rankings of the pages linking to page i.

For example,

$$r_1 = \alpha (r_2 + r_4 + r_5),$$

where α is a constant.

We get also

$$r_2 = \alpha (r_3 + r_4 + r_5)$$

$$r_3 = \alpha (r_1 + r_4)$$

$$r_4 = \alpha r_5$$

$$r_5 = \alpha r_3$$

In matrix notation:

$$\vec{r} = \alpha A \vec{r} \Rightarrow A\vec{r} = \frac{1}{\alpha} \vec{r}$$