EVOLUTION AND THE THEORY OF GAMES

STEFAN GERITZ & BRITTANY ROSE, HELSINKI, 2014

Exercises 28-11-2014

20. Consider the Iterated Stag Hunt: Each day, two players go out to hunt a stag (=male deer). They lay in ambush, waiting for the stag to come nearer. The first animal to come within reach, however, is a hare. The options for the players are: (S) ignore the hare and wait for the stag or (H) go for the hare and spoil the trap for the stag. If both players wait for the stag, they share the prize; if both players go for the hare they share the (smaller) prize; if only one of the players goes for the hare, he gets the hare with only half the probability, but he will not share the prize, and the other player will get nothing.

It takes two hunters to kill a stag, but one hunter can capture a hare. Moreover, the stag can defend itself, but the hare cannot. Killing the stag, therefore, is riskier for the hunters than killing the hare. The probability that both players are fit enough to go for a hunt the next day again therefore depends on the strategies.

Γ	S	H
S	$R + \delta\,\Gamma$	$\Delta\,\Gamma$
H	$r + \Delta\,\Gamma$	$r + \Delta\,\Gamma$

(symmetric game; payoff to row-player)

where R is half the expected value of the stag and r half the expected value of the hare, and where δ and Δ are the probabilities of a next round.

(a) Analyse the iterated Stag Hunt with $0 < r < R$ and $0 < \delta < \Delta < 1$.

As a next part of the exercise, suppose that if a stag is killed, the hunters can afford to skip one day of hunting and instead they spend at home. The probability of surviving a day at home is the same as for a day of hunting hare. If a hare is killed, the hunters have to go out the next day again. This modification of the game can be modelled with the following two sub-games:

Γ_1	S	H
S	$R + \delta\,\Gamma_2$	$\Delta\,\Gamma_1$
H	$r + \Delta\,\Gamma_1$	$r + \Delta\,\Gamma_1$

(symmetric game; payoff to row-player)

Γ_2	rest
rest	$\Delta\,\Gamma_1$

(symmetric game; payoff to row-player)

(b) Analyse the game $\Gamma = (\Gamma_1, \Gamma_2)$ with $0 < r < R$ and $0 < \delta < \Delta < 1$.

As the last part of the exercise, consider the Stag Hunt with three strategies: "every day a stag" (S) and "every day a hare" (H) and "one rest day after a day of successful stag hunting" (SR). If S goes with SR, then the latter takes a day off, while the former goes hunting a hare alone. This situation can be modelled by the three-stage game $\Gamma = (\Gamma_1, \Gamma_2, \Gamma_3)$ with

Γ_1	S	SR	H
S	$R + \delta\,\Gamma_1$	$R + \delta\,\Gamma_2$	$\Delta\,\Gamma_1$
SR	$R + \delta\,\Gamma_2$	$R + \delta\,\Gamma_3$	$\Delta\,\Gamma_1$
H	$r + \Delta\,\Gamma_1$	$r + \Delta\,\Gamma_1$	$r + \Delta\,\Gamma_1$

(symmetric game; payoff to row-player)

Γ_2	rest (SR-player)
hunt a hare (S-player)	$r + \Delta\,\Gamma_1,\ \Delta\,\Gamma_1$

(asymmetric game)

Γ_3	rest (SR-player)
rest (SR-player)	$\Delta\,\Gamma_1$

(symmetric game; payoff to row-player)

Notice that although Γ_2 is an asymmetric game, the players are given only one option each: "rest" for the SR-player and "hunt a hare" for the S-player.

(c) Analyse the game $\Gamma = (\Gamma_1, \Gamma_2, \Gamma_3)$ with $0 < r < R$ and $0 < \delta < \Delta < 1$.

21. Formulate the iterated prisoner's dilemma with strategies "tit for tat" (TFT) and "suspicious Pavlov" (sPav) as a multi-stage game. No need to solve the game; just reformulate as a multi-stage game.