University of Helsinki / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 05, 7.10.2013

N.B. The files mentioned in the exercises (if any) are available on the course homepage. The exam will be on Tuesday 15 th of October, 2013, 13-15

1. (a) Plot the functions $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-y^2)dy$ and $f(x) = \frac{1}{\sqrt{\pi}} \exp(-x^2)$ on the interval $[0, 3]$. Note that erf(x) is a built-in function of MATLAB. Find the root of the equation $f(x) = erf(x)$ on this interval.

(b) Show by change of variable that for $a, b \in R, a \neq 0, x_1 < x_2$,

$$\int_{x_1}^{x_2} \exp\left(-\frac{(x-b)^2}{(2a^2)}\right) dx = a\sqrt{\frac{\pi}{2}} \left(erf(\frac{x_2 - b}{a\sqrt{2}}) - erf(\frac{x_1 - b}{a\sqrt{2}}) \right) .$$

Verify this also by MATLAB experiments.

2. On the www-page is given the program `hpl052.m` which compares two methods of numerical integration, namely Riemann's sum and Simpson's Rule, over a rectangular region in the plane with the test function $f(x, y) = xy$. The program prints the error.

(a) Modify the program to use the function $g(x, y) = \sin(2x) * \cos(4 * y)$ and report the results.

(b) Write the code also for the Trapezoid Rule and the MATLAB built-in function `dblquad` and report the error. Provide an order or preference of the methods based on the accuracy of each method.

3. Use MATLAB to generate a picture of the Julia set of the iteration $z \mapsto z^2 + a, a = 0.3 - i * 0.2$.

4. Suppose that A is a non-singular $n \times n$ matrix with columns $A^{(j)}, j = 1, ..., n$, and x and b are $n \times 1$ vectors. By Cramer's Rule, the solution to $Ax = b$ is given by

$$x_j = ((\det(A))^{-1}) \det(C_j), C_j = [A^{(1)} A^{(2)} ... A^{(j-1)} b A^{(j+1)} ... A^{(n)}] .$$

Verify this procedure with MATLAB tests for small n. For how big values of n this is a reasonable procedure?

5. The daily measurement data of a the body temperature of a patient are stored in files a1.txt,..., a7.txt in the format one measurement/line.

FILE: ~/mme13/demo13/d05.tex — 30. syyskuuta 2013 (klo 10.11).

Write a program that reads the measurements and plots a histogram (the command bar and hist may be useful here) of the results and computes the mean temperature.

6. Theorem 1.2.2 on p. 12 of P. Borwein-T. Erdélyi: Polynomials and Polynomial Inequalities Springer-Verlag, 1995 states that if $p(z) = a_n z^n + a_{n-1} z^{n-1} + \ldots + a_0$ and $a_0 \geq a_1 \geq \ldots \geq a_n > 0$ then all zeros of p lies outside the open unit disk. Verify experimentally this statement by generating random coefficients a_j and by plotting the roots in the plane.