Exercises 4

(1.)

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 1 & \frac{1}{2} \\ 3 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

stability: calculate the eigenvalues by solving

$$f(\lambda) = \det(A - \lambda I) = 0$$

$$f(\lambda) = \det(A - \lambda I) = \begin{vmatrix} 1-\lambda & \frac{1}{2} \\ 3 & \frac{1}{2}-\lambda \end{vmatrix} = (1-\lambda)(\tfrac{1}{2}-\lambda) - \frac{3}{2}$$

$$= \frac{1}{2} - \frac{1}{2}\lambda - \lambda + \lambda^2 - \frac{3}{2} = \lambda^2 - \frac{3}{2}\lambda - 1 = (\lambda + \tfrac{1}{2})(\lambda - 2)$$

$$\Rightarrow \lambda_1 = -\frac{1}{2} \quad \lambda_2 = 2$$

Because atleast one eigenvalue is > 1 $\Rightarrow$ the origin $(x, y) = (0,0)$ is unstable.

(2.)

$$\begin{cases} p' = p \dfrac{V_1}{\overline{V}} \\ q' = q \dfrac{V_2}{\overline{V}} \end{cases} \quad \text{where} \quad V_i = \sum_j V_{ij} p_j \ , \ V_{ij} = V_{ij}(p)$$

$$\overline{V} = \sum_{ij} V_{ij} p_i p_j$$

stability of $p = 0$ is determined by:

$$\left.\frac{\partial p'}{\partial p}\right|_{p=0} = \frac{V_1(0)}{\overline{V}(0)} + \left[p \frac{\partial V_1(p)/\overline{V}(p)}{\partial p} \right]_{p=0} = \frac{V_{12}(p=0)}{V_{22}(p=0)}$$

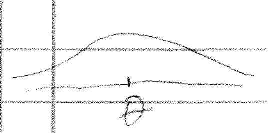
Stability of $p = 1 \Rightarrow q = 0$ is determined by:

$$\frac{\partial q'}{\partial q} = \frac{V_2(0)}{\overline{V}(0)} + \left[q \frac{\partial V_2(p)/\overline{V}(p)}{\partial q} \right]_{q=0} = \frac{V_{12}(q=0)}{V_{11}(q=0)}$$

Prot. Coex.: $\dfrac{V_{12}(p=0)}{V_{22}(p=0)} > 1$ & $\dfrac{V_{12}(q=0)}{V_{22}(q=0)} > 1$

(3.) $V_{ij} = S_{ij}\,\psi_{ij}$

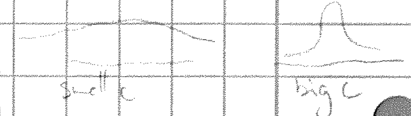
$S_{ij} = S_0 \exp[-s(\phi_{ij}-\theta)^2]$ | fi-selection



$\psi_{ij} = \rho - C_{ij}$ | fd-selection

$C_{ij} = \sum_{kl} v_{ij,kl} P_{kl}$ $\left(= \sum v_{ij,kl}\, p_k p_l\right)$

$v_{ij,kl} = v_0 \exp[-c(\phi_{ij}-\phi_{kl})^2]$



$p' = p\,\frac{V_1}{\bar{V}}$, where V_{ij} is given above

ϕ_{ij} is the phenotype of A_iA_j

Now, suppose $x_1=-1$, $x_2=1$ $\Rightarrow$ $\phi_{11}=-1$, $\phi_{12}=0$ $\phi_{22}=1$
& $\theta = 0$ (note, A_1A_2 utilize the fi-resource the best)

(a) $V_{ij} = V_{ij}(p)$ $\Rightarrow$ $p=0$ & $p=1$ unstable if $\frac{V_{12}(0)}{V_{22}(0)} > 1$ & $\frac{V_{12}(1)}{V_{11}(1)} > 1$

$V_{12}(0) = S_0 \exp[-s\cdot 0^2]\,(\rho - v_{12,22})$

$= S_0\,(\rho - v_0 \exp[-c])$

$V_{22}(0) = S_0 \exp[-s]\,(\rho - v_0 \exp[-c\cdot 0]) = S_0 \exp(-s)(\rho - v_0)$

$\Rightarrow \quad \frac{V_{12}(0)}{V_{22}(0)} = \frac{1}{\rho - v_0} \exp(s)\,(\rho - v_0 \exp(-c)) > 1 \qquad (1)$

$V_{12}(1) = S_0\,(\rho - v_0 \exp[-c])$

$V_{11}(1) = S_0 \exp[-s]\,(\rho - v_0 \exp[-c\cdot 0])$

same invasion condition as above : PC if (1) holds

(b) $\dfrac{V_{12}(0)}{V_{22}(0)}$ is an increasing function of c

$\Rightarrow$ increasing c protection is "easier"

(c) $\dfrac{V_{12}(0)}{V_{22}(0)}$ is an increasing function of s (narrower resource distr. ~ heteros at greater adv.)

$\Rightarrow$ increasing s protection is "easier"