Exercise 6

1. I have attached a paper by Lynch 2010, which gives a nice account of mutation rates.

2. Equation (pure mutation equation) for two alleles with rates $\mu_{12} = \mu$ and $\mu_{21} = \nu$ is

$$p' = p(1-\mu) + (1-p)\nu$$

where p is the freq. of A_1 and $1-p$ the freq. of A_2.

The equilibrium is then given by

$$\hat{p} = \frac{\nu}{\mu+\nu}$$

If $\nu = \frac{1}{100}\mu$, then

$$\hat{p} = \frac{\frac{1}{100}\mu}{\mu + \frac{1}{100}\mu} = \frac{\frac{1}{100}}{\frac{101}{100}} = \frac{1}{101} \approx 0.01$$

3. An allele is said to be protected when it can increase in frequency when rare.

If denoting the frequency of A_1 with p and the frequency of A_2 with $q=1-p$, the allele A_2 is rare in the neighborhood of the point $p=1$.

Note, that in the mutation models, the trivial states $p=0$, $p=1$ are not in general equilibria, as the frequency of an allele can increase even if the allele is initially completely absent. This is, because the resident allele may mutate and produce these initially "missing" types.

The dynamical system for the model in question is

$$(1) \quad p' = p\frac{W_1}{\overline{W}} + \frac{1}{\overline{W}}\left[pW_1\mu_{11} - pW_1\mu_{11} + qW_2\mu_{21} - pW_1\mu_{12}\right]$$

$$\overset{\mu_{11}=0,\ \mu_{21}=0}{=} p\,\frac{W_1(1-\mu_{12})}{\overline{W}}, \quad \text{where } \overline{W}=\overline{W}(p)=pW_1+(1-p)W_2$$

Alternatively, we could write the dynamics in terms of q.

Solving (1) for equilibria $p'=p$, we get that $\hat{p}^{(1)}=0$ and $\hat{p}^{(2)} = 1 + \mu_{12}\dfrac{W_1}{W_2-W_1}$

Thus, $\hat{p}=1$ is NOT an equilibrium iff $\mu_{12}>0$

This fact simplifies the analysis, since now it is enough to show that if initially allele A_2 is not present ($p=1$, i.e. $q=1-p=0$), then in the next generation it always has a positive frequency, i.e. $p'<1$ (or $q'>0$).

Inserting $p=1$ into (1), we get that

$$p' = 1 \cdot \frac{w_1(1-\mu_{12})}{w_1} = 1-\mu_{12}.$$

Thus, $p'<1$ for $\mu_{12}>0$

$\Rightarrow$ <u>A_2 is always protected</u>

(Note that this is the case even if A_2 is being selected against, $w_2<w_1$.

It can also be shown that for $w_1>w_2$ and $\frac{w_1}{w_2}(1-\mu_{12})>1$ there exists a stable <u>interior</u> equilibrium where both alleles can coexist)