15.11.2012

Mathematical theory of population genetics

Exercises 8.

1. (2 points) Show that in the Wright-Fisher model the variance of each Bernoulli trial (i.e. each reproduction event, where either A_1 or A_2 is passed on to the offspring) is $p(1-p)$, where p is the probability of success (i.e. the frequency of A_1 alleles amongst parents).

2. (2 points) Show that in the Moran model the variance of the number of A_1 alleles in the next generation is $2p(1-p)$, where p denotes the frequency of A_1 in this generation (i.e. show that $\mathrm{Var}[K_1] = 2p(1-p)$, where $pN = K_0$).

3. Show that the expected heterozygosity $\mathrm{E}[H_1]$ in the
   (a) (4 points) Wright-Fisher model is $H_0(1-\frac{1}{N})$.
   (b) (4 points) Moran model is $H_0(1-\frac{2}{N^2})$.

4. (4 points) Considering Wright-Fisher or Moran model, show that knowing the distribution of the initial state K_0, then

$$\mathrm{E}[K_{t+1}] = \mathrm{E}[K_t] = \cdots = \mathrm{E}[K_0]. \tag{1}$$