Mathematical theory of population genetics

Exercises 7.

1. (3 points) Show that with two loci, arbitrary number of alleles, random mating and with no evolutionary processes (no selection, mutation etc.), the allele frequencies are conserved.

2. (3 points) Show that $D = x_1 x_4 - x_2 x_3 = D_{11} = -D_{12} = -D_{21} = D_{22}$ (see also the Lecture notes).

3. (4 points) Suppose that fitnesses of genotypes are determined additively from fitnesses at individual loci. Then the fitness matrix is

$$\begin{pmatrix} a_{11} + b_{11} & a_{11} + b_{12} & a_{11} + b_{22} \\ a_{12} + b_{11} & a_{12} + b_{12} & a_{12} + b_{22} \\ a_{22} + b_{11} & a_{22} + b_{12} & a_{22} + b_{22} \end{pmatrix} \tag{1}$$

where the fitness contributions of the one-locus genotypes are a_{ij} for $A_i A_j$ and b_{ij} for $B_i B_j$, $i, j = 1, 2$. Show that the mean fitness of the population $\overline{W}$ is independent of gamete frequencies x_i, $i = 1, 2, 3, 4$.