18.10.2012

Mathematical theory of population genetics

Exercises 5.

1. (3 points) Dispersal is called *reciprocal* if the number of individuals that migrate from deme α to deme β equals the number that migrate from β to α:
$$c^*_\alpha \tilde{m}_{\alpha\beta} = c^*_\beta \tilde{m}_{\beta\alpha}. \tag{1}$$
Show, that if this holds for all pairs of demes then $\tilde{m}_{\alpha\beta} = m_{\alpha\beta}$.

2. *Random outbreeding and site homing* (Deakin 1966). The migration pattern is defined as follows:
$$m_{\alpha\beta} = \mu c^*_\beta \quad \text{if } \alpha \neq \beta, \tag{2}$$
$$m_{\alpha\alpha} = 1 - \mu + \mu c^*_\alpha, \tag{3}$$
where $\mu \in [0,1]$ is a constant that describes the proportion of outbreeding individuals. These individuals leave their deme of origin and are dispersed randomly over all other demes according to the deme sizes. ($\mu = 0$, no migration; $\mu = 1$, all individuals outbreed).
(a) (4 points) Show that $c^0_\beta = c^*_\beta$.
(b) (2 points) Show that migration is reciprocal, i.e. $c^*_\alpha \tilde{m}_{\alpha\beta} = c^*_\beta \tilde{m}_{\beta\alpha}$.

3. (4 points) In the soft-selection model of Levene (1953), the dynamical equation can be written as
$$p'_i = p_i \sum_\alpha c_\alpha \frac{W_{i,\alpha}}{\bar{W}_\alpha}. \tag{4}$$
Consider two demes and two alleles, such that the relative fitnesses in deme 1 are $W_{11,1} = 1, W_{12,1} = 1 - hs, W_{22,1} = 1 - s$ and in deme 2 $W_{11,2} = 1 - s, W_{12,2} = 1 - hs, W_{22,2} = 1$. Supposing the demes are of equal size, give a condition for protected coexistence.

References

[1] Deakin, M.A.B. 1966. Sufficient conditions for genetic polymorphism. Am. Nat. 100: 690–692.

[2] Levene, H. 1953. Genetic equilibrium when more than one ecological niche is available. Am. Nat. 87: 331–333.