

Remark. Use of an abstract page of the size A4 is allowed to a candidate.

1. Solve the differential equation

$$(\cos x + \sin y) + (\sin y + x \cos y)y' = 0.$$

An implicit solution is sufficient.

2. Solve the initial value problem

$$x^3y' - 2x^2y = 4, \quad y(-1) = 1,$$

and give also a maximal solution interval of it.

3. Solve the differential equation

$$2y'' + 4y' + 4y = xe^x.$$

4. We expect from a behavior of a function $x = x(t)$ as follows: A factor makes the function to increase at the rate proportional to a square of a value at present, a as a proportional constant, and another factor in turn makes it to decrease at the rate proportional to a cubic value, b as a proportional constant. Additionally suppose $a > b > 0$.

(a) (2 p) Knowing that write a differential equation for the function $x(t)$. Of what basic type is it?

(b) (4 p) Let $x(t)$ be a particular solution to the equation satisfying the initial condition $x(0) = 1$. What can you tell about its maximal solution interval and limits when $t \rightarrow \pm\infty$? It is not necessary to solve the equation.