Differential Equation I

Exercise 5, fall 2012

1. Study by the Wronskian determinant, which of the pairs y_1 and y_2 can in principle form a fundamental solution set on the whole $\mathbf{R}$ for a homogeneous equation of second order, when
(a) $y_1(x) = x^3$ and $y_2(x) = x$, (b) $y_1(x) = \sin 2x$ and $y_2(x) = \cos 2x$,
(c) $y_1(x) = \sin 2x$ and $y_2(x) = \cos x$.

2. Solve the equations

$$(a) \quad 3y'' + 2y' + y = 0, \quad (b) \quad \ddot{x} - 4\dot{x} + 4x = 0.$$

3. Solve the equation

$$\ddot{x} + 9x = 4t - 2 + e^{-3t}$$

by using an appropriate direct try (the method of undetermined coefficients).

4. Solve the equation

$$\ddot{x} - 9x = e^{-3t}$$

by variation. So, what would have been an appropriate form for a direct try?

5. Study on the interval $]0, \infty[$ the homogeneous equation

$$4x^2y'' + 4xy' - y = 0.$$

(a) Find by an appropriate direct try a fundamental solution pair on that.
(b) Solve the equation on that by changing appropriately the free variable x.

6. The homogeneous equation

$$(x-2)y'' - (4x-7)y' + (4x-6)y = 0$$

has a solution e^{2x} on $\mathbf{R}$. Find another independent solution by using the method of variation and give also a general solution on $\mathbf{R}$. Does that directly give all the solutions?