STOCHASTIC POPULATION MODELS

EXERCISES 9

9.

Consider the following variation on the individual-level model from section 1.9, where X denotes an adult individual inhabiting a territory, Y a juvenile juvenile individual, Z an adult individual searching for a free territory, and T a free territory. Suppose the individual-level dynamics are give by the reaction network

$$\begin{array}{rcll} \mathrm{X} & \xrightarrow{\alpha} & \mathrm{X}+\mathrm{Y} & \text{(reproduction)} \\ \mathrm{Y} & \overset{\varphi}{\Longrightarrow} & \mathrm{Z} & \text{(maturation)} \\ \mathrm{Z}+\mathrm{T} & \xrightarrow{\beta} & \mathrm{X} & \text{(colonisation of territory)} \\ \mathrm{X} & \xrightarrow{\gamma} & \mathrm{T} & \text{(adult death in territory)} \\ \mathrm{Y} & \xrightarrow{\varepsilon} & \dagger & \text{(juvenile death)} \\ \mathrm{Z} & \xrightarrow{\delta} & \dagger & \text{(adult death outside territory)} \end{array}$$

Here $\mathrm{Y} \overset{\varphi}{\Longrightarrow} \mathrm{Z}$ is to be interpreted as in section 3.1 of the lecture notes, i.e., as a developmental delay where φ is the probability density of the length of the juvenile period given that the juvenile survives till maturation.

(a) Give the corresponding population equations as a closed system of two differential equations: one for the population density X of individuals occupying a territory, and one for the population density Z of individuals searching for a territory.

(b) Assume that reproduction by adults inside a territory as well as adult death outside a territory are both fast processes. Reduce the system to a single DDE for X only. Take $\varphi(t) = \delta_{\mathrm{Dirac}}(t-\tau)$ for given $\tau > 0$. What does this mean in terms of the processes on the individual level? Compare the result with the delayed logistic in equation (5) of section 3.1 of the lecture notes.

(c) Perform a linear stability analysis for the trivial equilibrium $X = 0$. You can use the figures derived in section 3.1 of the lecture notes, but beware that not all values of $a\tau$ and $b\tau$ can be realised in the present model.

(d) When does the DDE in **(b)** have a positive equilibrium $\overline{X} > 0$. Perform a linear stability analysis for $\overline{X}$ when it exists. You can use the figures derived in section 3.1 of the lecture notes, but beware that not all values of $a\tau$ and $b\tau$ can be realised in the present model.

(e) Calculate the transfer function for fluctuations in the juvenile death rate when the population density X is close to a positive equilibrium $\overline{X}$.