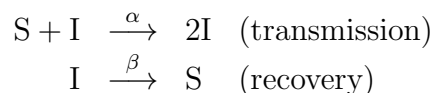


STOCHASTIC POPULATION MODELS

EXERCISES 5-8

5.

Consider the epidemiological model of the last exercise based on the processes



Give the transfer function $T(\omega)$ for a seasonally varying infection rate α . Calculate the maximum gain G_m and the cutoff frequency ω_c . How does the cutoff frequency depend on the average infection rate? Express the cutoff frequency in units of *cycles per mean recovery time* (β^{-1}). What does this tell you about absence (or presence) of seasonality of the prevalence of the disease in relation to the average length of an infection?

6.

Consider the resource-consumer model

$$\begin{cases} \frac{d}{dt}R = R_0 - \alpha R - \beta RC & \text{(resource)} \\ \frac{d}{dt}C = \gamma\beta RC - \alpha C - \delta C^2 & \text{(consumer)} \end{cases}$$

Can you interpret the different terms and parameters? Calculate the equilibria and determine their stability. For the positive equilibrium, give the transfer functions $T_R(\omega)$ and $T_C(\omega)$ for, respectively, the resource and the consumer if the R_0 is seasonally varying. Under what conditions is there a resonance peak?

7.

Let \tilde{f} and \tilde{h} denote the Fourier transforms of, respectively, f and h , and show that:

- (a) the Fourier transform and its inverse are linear operators,
- (b) $\tilde{\tilde{f}}(t) = 2\pi f(-t)$,
- (c) $\widetilde{\left(\frac{d}{dt}f\right)}(\omega) = i\omega\tilde{f}(\omega)$,
- (d) $\frac{d}{d\omega}\tilde{f}(\omega) = -i\widetilde{(tf)}(\omega)$,
- (e) $\tilde{f}_\tau(\omega) = e^{-i\omega\tau}\tilde{f}(\omega)$ where $f_\tau(t) := f(t - \tau)$,
- (f) $\widetilde{(f * h)}(\omega) = \tilde{f}(\omega)\tilde{h}(\omega)$ where $(f * h)(t) := \int_{-\infty}^{+\infty} f(\tau)h(t - \tau)d\tau$,
- (g) $\widetilde{(fh)}(\omega) = \frac{1}{2\pi}(\tilde{f} * \tilde{h})(-\omega)$,
- (h) $\int_{-\infty}^{+\infty} f(t)\tilde{h}(t)dt = \int_{-\infty}^{+\infty} \tilde{f}(t)h(t)dt$.

8.

Let δ denote the Dirac-delta distribution, and show that:

- (a) $\widetilde{(t^n)}(\omega) = 2\pi n! \delta(\omega)/(i\omega)^n$ for $n = 0, 1, \dots$
- (b) $\widetilde{e^{i\omega_0 t}}(\omega) = 2\pi \delta(\omega - \omega_0)$
- (c) $\widetilde{\cos(\omega_0 t)}(\omega) = \pi \delta(\omega - \omega_0) + \pi \delta(\omega + \omega_0)$