**GEOMETRIC MEASURE THEORY AND SINGULAR INTEGRALS: LIST OF TOPICS COVERED ON THE LECTURES**

The references are to Tolsa's book. The first part of the course by H. Martikainen was 13 lectures, the contents of which are summarised below.

(1) Lecture 1 (Tue 17 Jan): Parts of section 3.2 covering the definition of Menger curvature, Proposition 3.1 (basic identities for the Menger curvature), Proposition 3.2 (the magic formula for the Menger curvature), definition of the curvature of a measure, definition of the Cauchy operator, statement of Proposition 3.3 (connection between curvature and Cauchy).
(2) Lecture 2 (Fri 20 Jan): Proof of Proposition 3.3, the statement of the $T1$ theorem (Theorem 3.5) for the Cauchy operator with parts of the proof (we will prove (b) implies (a) later), the definition of complex measures and $M(\mathbb{R}^d)$, definition of the centered maximal function $M_\mu$ (page 51) and discussion of some boundedness concepts (what it means that $M_\mu \colon M(\mathbb{R}^d) \to L^{1,\infty}(\mu)$ boundedly).
(3) Lecture 3 (Tue 24 Jan): Proved the boundedness properties for the centered maximal function $M_\mu$, discussed various covering theorems (the $5r$-covering theorem, Theorem 2.2, and the Besicovitch covering theorem, Theorem 2.3), defined various other maximal functions including the modified non-centered maximal function $\widetilde{M}_\mu$ and the $n$-dimensional radial maximal function $M_{R,n}$, proved their boundedness properties (essentially Theorem 2.6 modified slightly to use $\widetilde{M}_\mu$ instead of $N_\mu$), defined doubling balls and cubes and proved the existence of large doubling balls (beginning of Section 2.4), stated Lemma 2.8 but postponed the proof to the next lecture.
(4) Lecture 4 (Fri 27 Jan): Finished the proof of Lemma 2.8 (existence of small doubling cubes), stated the definitions of standard kernels and SIO (Section 2.2), stated the definitions concerning maximal truncations (Section 2.8), started proving Cotlar's inequality (Theorem 2.18).
(5) Lecture 5 (Tue 31 Jan): Finished the proof of Cotlar's inequality (Theorem 2.18) and proved the basic boundedness properties of $T_{\mu,*}$ (Theorem 2.21).
(6) Lecture 6 (Fri 3 Feb): Finished proving that the weak type bound for $T_\delta$ implies the same for $T_{*,\delta}$. This means that we completed the proof of Theorem 2.21 assuming the fact that the $L^2(\mu)$ boundedness of $T_{\mu,\delta}$ implies the weak type bound $M(\mathbb{R}^d) \to L^{1,\infty}(\mu)$ for $T_\delta$. Moved to proving this. Started proving the non-homogeneous Calderón–Zygmund decomposition (Lemma 2.14) – finished the part a. As background info talked about absolute continuity and Radon–Nikodym.
(7) Lecture 7 (Tue 7 Feb): Finished the proof of Lemma 2.14. Formulated a separate example of how to use Lemma 2.14 in practice:

$$\nu = g\,d\mu + \sum_i \beta_i$$

with specific properties of $g \in L^2(\mu)$ and the cancellative complex measures $\beta_i$.

(8) Lecture 8 (Fri 10 Feb): Proved that the $L^2(\mu)$ boundedness of $T_{\mu,\delta}$ implies the weak type bound $M(\mathbb{R}^d) \to L^{1,\infty}(\mu)$ for $T_\delta$.This is Theorem 2.16.
(9) Lecture 9 (Tue 14 Feb): We moved to the big piece method/good lambda method (Section 2.9). Proved Lemma 2.23 i.e. the non-homogeneous Whitney decomposition in detail. Started proving Theorem 2.22 i.e. the good lambda method.
(10) Lecture 10 (Fri 17 Feb): Finished the proof of Theorem 2.22.
(11) Lecture 11 (Tue 21 Feb): Returned to the Cauchy transform: We proved the remaining part of the $T1$ for Cauchy i.e. that (b) implies (a) in Theorem 3.5.
(12) Lecture 12 (Fri 24 Feb): Proved Lemma 3.9 and using this proved that

$$\int_I \int_I \int_I c((x,(A(x)),(y,(A(y)),(z,(A(z)))^2 \, dx \, dy \, dz \lesssim_{\mathrm{Lip}(A)} \ell(I) \tag{0.1}$$

for all intervals $I \subset \mathbb{R}$, if $A\colon \mathbb{R} \to \mathbb{R}$ is Lipschitz.
(13) Lecture 13 (Tue 28 Feb): Proved that (0.1) implies that the $T1$ condition (c) of Theorem 3.5 holds in the case that $\mu$ is $\mathcal{H}^1$ restricted to $\Gamma = \mathrm{graph}(A)$, $A\colon \mathbb{R} \to \mathbb{R}$ Lipschitz. This showed Theorem 3.11. Formulated the property BPLG i.e. big pieces of Lipschitz graphs (and also (BP)$^2$LG) for sets $E \subset \mathbb{C}$. Showed that Cauchy is bounded also on such sets. Showed that ADR curves have BPLG (Lemma 3.13). In particular, showed Theorem 3.12.