Forcing

Exercise 6

1. For any $p \in P$, $p : \alpha \to \kappa$, and $\beta < \kappa$, if $\beta \in \mathrm{rng}(p)$ then $p \Vdash \hat{\beta} \in \mathrm{rng}(\bigcup \dot{G})$ and otherwise $q = p \cup \{(\alpha, \beta)\} \in P$ and $q \Vdash \hat{\beta} \in \mathrm{rng}(\bigcup \dot{G})$ and $q \leq p$.

2. By 6.4 we may assume that $V \models CH$. Let $\kappa = 2^{\omega_1}$ and P be the set of all $p : \alpha \to \kappa$ s.t. $\alpha < \omega_2$ ordered by inverse subset relation. Clearly P is ω_2-closed and by Δ-lemma one can see that P has κ^+-cc. Thus P preserves cardinals $\leq \omega_2$ or $> \kappa$ and does not add subsets to ω nor to ω_1 and forces that $|\kappa| = \omega_2$.

3. By 4.11 we may assume that $V \models 2^{\omega} \geq \omega_2$. Then force with the p.o.-set from Ex 2.

4 Just check the definitions

5. Let $D \subseteq P$ be dense and let

$D^* = \{ (p, \tau) \in P * Q \mid p \in D \}$.

By Ex 7.2 (iii) D^* is dense in $P*Q$.

Thus there is $(p, \tau) \in D^* \cap K$.

But then $(p, 1) \in K$ and so $p \in D \cap i^{-1}(K)$.

6. (iv) if $q \restriction \gamma \perp q' \restriction \gamma$ then clearly $q \perp q'$.

So we suppose that there is $r \leq q \restriction \gamma, q' \restriction \gamma$.

Let s be such that for all $\beta < \gamma$,

$s(\beta) = r(\beta)$ and for $\gamma \leq \beta < \alpha$ let

$s(\beta) = q(\beta)$ if $q(\beta) \neq 1$ and

$s(\beta) = q'(\beta)$ otherwise. Clearly s is

as wanted.

(vi) Just check the definitions.