Forcing

Exercise 4

1. Start by applying Theorem 1.2.3 to the following class function $G: V \to V$.

If (*) below holds then $G(x)$ is a function from P to 2 s.t. for all $p \in P$ $G(x)(p) = 1$ iff

(a) for all $q \leq p$ and $(\tau', s) \in \tau$, if $q \leq s$, then there are $r \leq q$ and $(\sigma', t) \in \sigma$ s.t. $r \leq t$ and $x(\tau', \sigma')(r) = 1$

and

(b) for all $q \leq p$ and $(\sigma', t) \in \sigma$, if $q \leq t$, then there are $r \leq q$ and $(\tau', s) \in \tau$ s.t. $r \leq s$ and $x(\tau', \sigma')(r) = 1$

(See Definition 3.2)

(*): There are P-names τ and σ s.t.

$$\mathrm{dom}(x) = \{(\tau', \sigma') \mid \tau', \sigma' \ P\text{-names and } \tau' \in TC(\tau) \text{ and } \sigma' \in TC(\sigma')\}$$

and for all $(\tau', \sigma') \in \mathrm{dom}(x)$,

$x(\tau', \sigma')$ is a function from P to 2.

If (*) is not true let $G(x) = \emptyset$.

2. Just check the definitions

3. By induction on φ.

4 If $\delta_G, \varrho_G \in V[G]$, let
$\tau = \{(\sigma, 1), (\varrho, 1)\}$. Then $\tau_G = \{\delta_G, \varrho_G\}$.

5. Let $A \subseteq P$ be maximal s.t. for all
$q, r \in A$, if $q \neq r$ then $q \perp r$,
for all $q \in A$, $q \leq p$ or $q \perp p$,
and if $q \leq p$, then there is τ_q s.t.
$q \Vdash \varphi(\tau_q, \tau_1, \ldots, \tau_n)$. Then A is
a maximal antichain. Let $\tau =$
$\{(\delta, q) \mid$ for some $r \in A$, ~~...~~ $r \leq p$,
there is $(\delta, s) \in \tau_r$ s.t. $q \leq r, s\}$
Then $\tau_G = (\tau_q)_G$ if $q \leq p$ and $q \in G \cap A$.
Thus τ is as wanted

6. W.o.l.g. we may assume that if
~~...~~ σ and τ are P-names, $\tau \in C$
and $\delta \in TC(\tau)$ then $\delta \in C$. Then
just apply theorem 1.2.3.