CT imaging of hard materials with beam hardening
A Comparison of Back Projection and Tikhonov method

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Introduction

The purpose of this study was to investigate the competence of filtered back projection and Tikhonov regularization in CT imaging of hard materials. Our main goal was to test suitability of these methods in handeling beam hardening. Due to the complexity of imaging metals and other hard materials, we have chosen a SIM-card and a stone as sample materials.

Materials

We used in this examination a typical mini-SIM with metal plate. This plate is the most valuable part of any SIM card, because in contains all the electronics of the card.
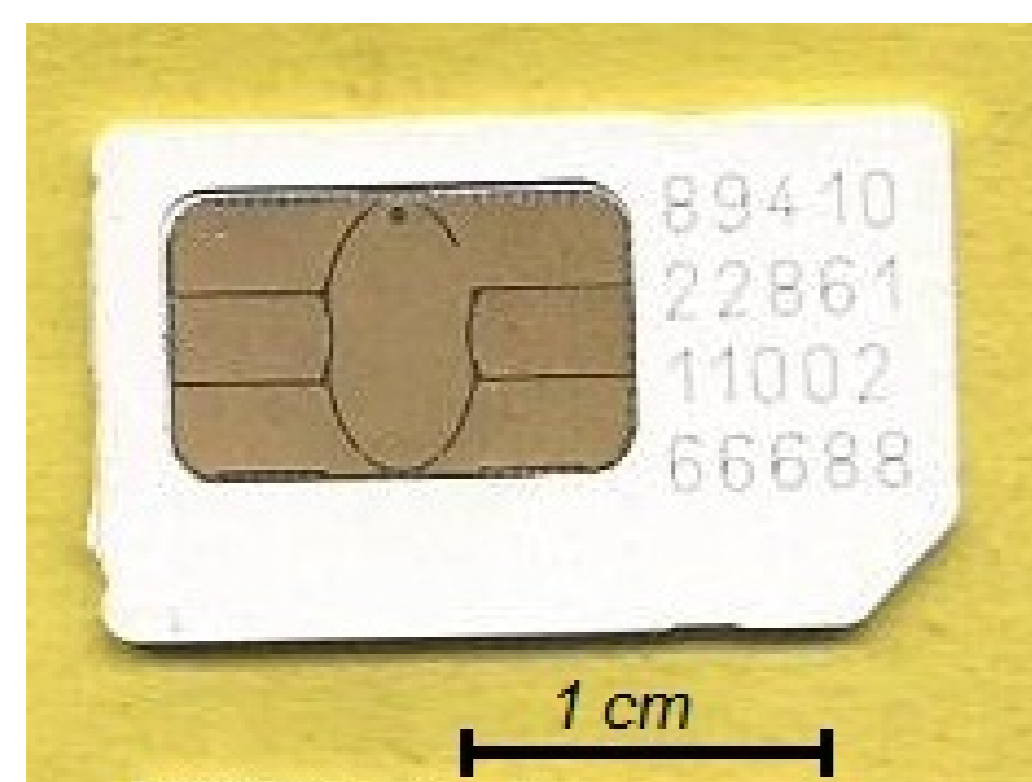

Figure 1: Used SIM-card with scale.

Our other sample was a porous limestone, which has most probably been formed from a petrifyed coral. Stones are in general quite dense, so they have similar imaging properties as metals and other hard materials.
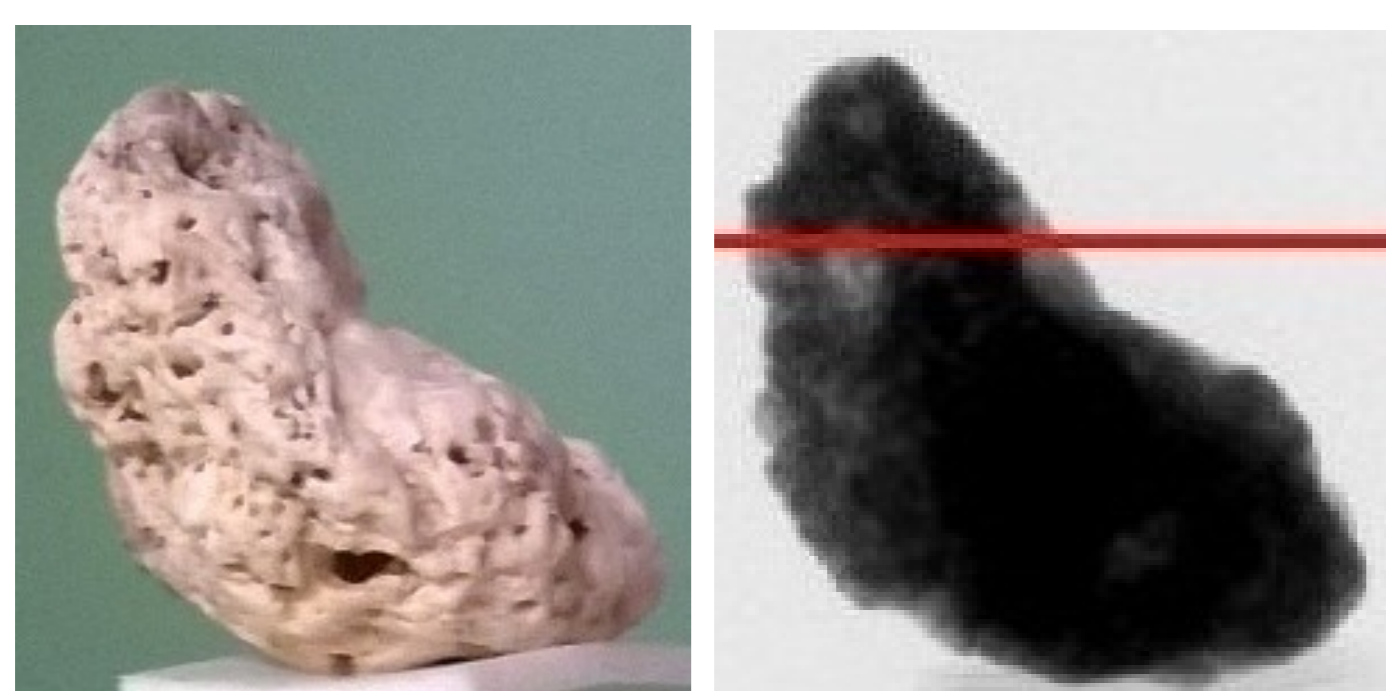

Figure 2: Original stone on the left, X-ray image with the examination plane on the right.

Methods

We took CT tomography images of both samples in 20 projection directions. As the X-ray beam passes through dense material, such as metal, the energy of the beam increases, because lower energy photons are absorbed more rapidly than the higher-energy photons. This shift of the X-ray spectrum to higher effective energies is called beam hardening. The phenomenon causes broadening and brightening of the edges especially in dense materials. We produced the reconstruction images with filtered back projection and Tikhonov regularization. A parallel beam assumption was used in all reconstructions.

Filtered back projection

The measurement data is actually a Radon transform of the sample, and filtered back projection is based on the inverse Radon transform. We used back projection with a simple ramp filter.
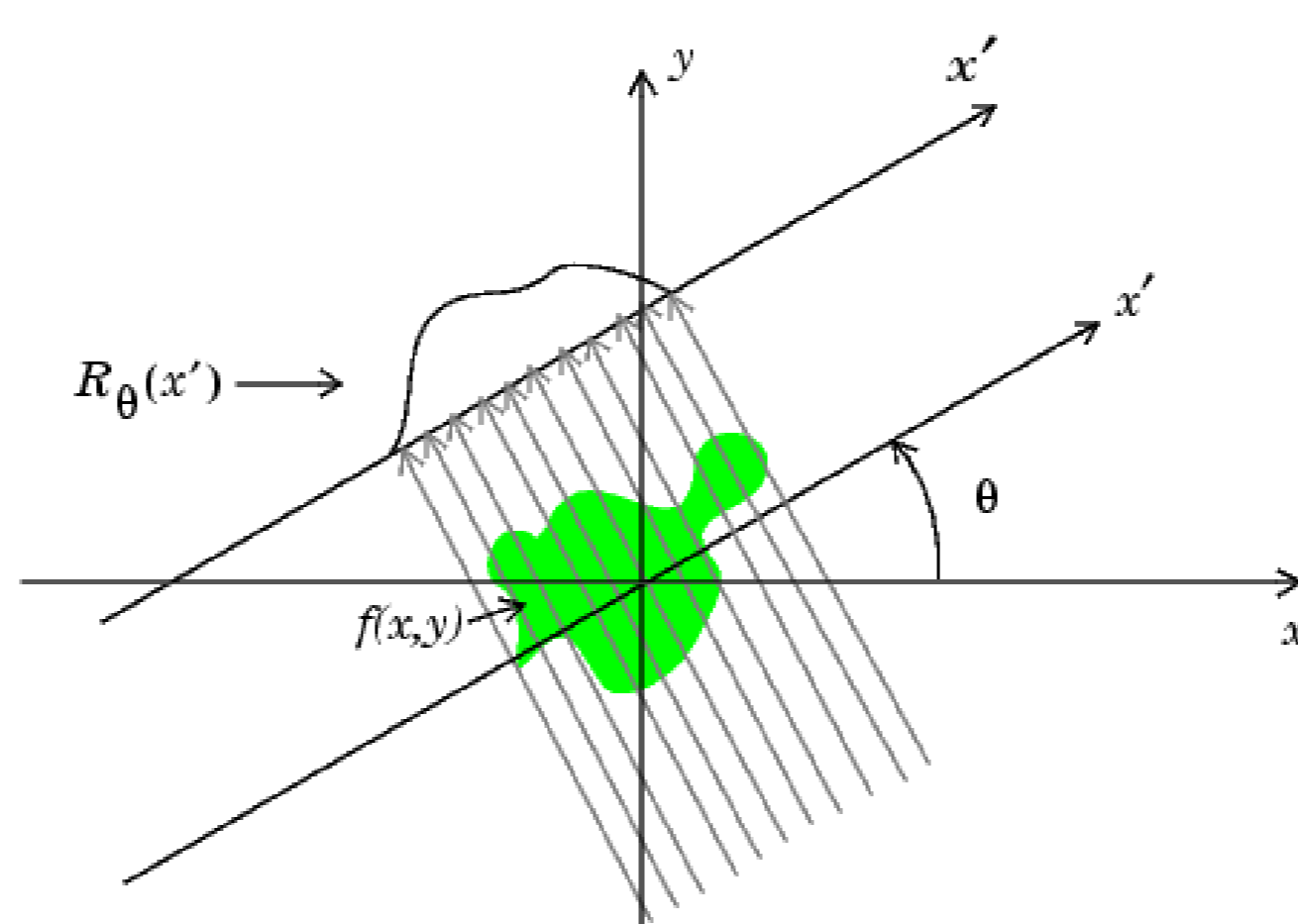

Figure 3: Principle of Radon transform with parallel beams.

Tikhonov regularization

Tikhonov regularization is based on minimizing

$$\|A\mathbf{f} - \mathbf{m}\|_2^2 + \alpha \|L\mathbf{f}\|_2^2. \qquad (1)$$

Now the best solution T_α is obtained by solving equation

$$(A^T A + \alpha L^T L)\mathbf{f} = \mathbf{A^T m}. \qquad (2)$$

The regularization parameter α was chosen via L-curve method. For this we computed solutions with 30 α-values, and plotted $\|T_{alpha}\|_2$ with respect to $\|AT_{alpha} - m\|_2$ on a logarithmic scale. The best α-value was found on the solution nearest the angle of the L-chaped curve. In practice α was chosen by taking the point with the smallest positive curvature.

We found the solution iteratively on a matrix-free way by using conjugate gradient method. There the solution is obtained by altering an initial guess, obtained from simple back projection. The direction of the alteration is conjugate to the gradient. This method is really fast, since it converges with only a quite small number of steps. We stopped the iteration when the largest change to the solution was smaller than a pre-defined accuracy parameter.

Results

We used Tikhonov regularization and filtered back projection to compute the reconstuction of CT images. These results are visualized in the figures below.
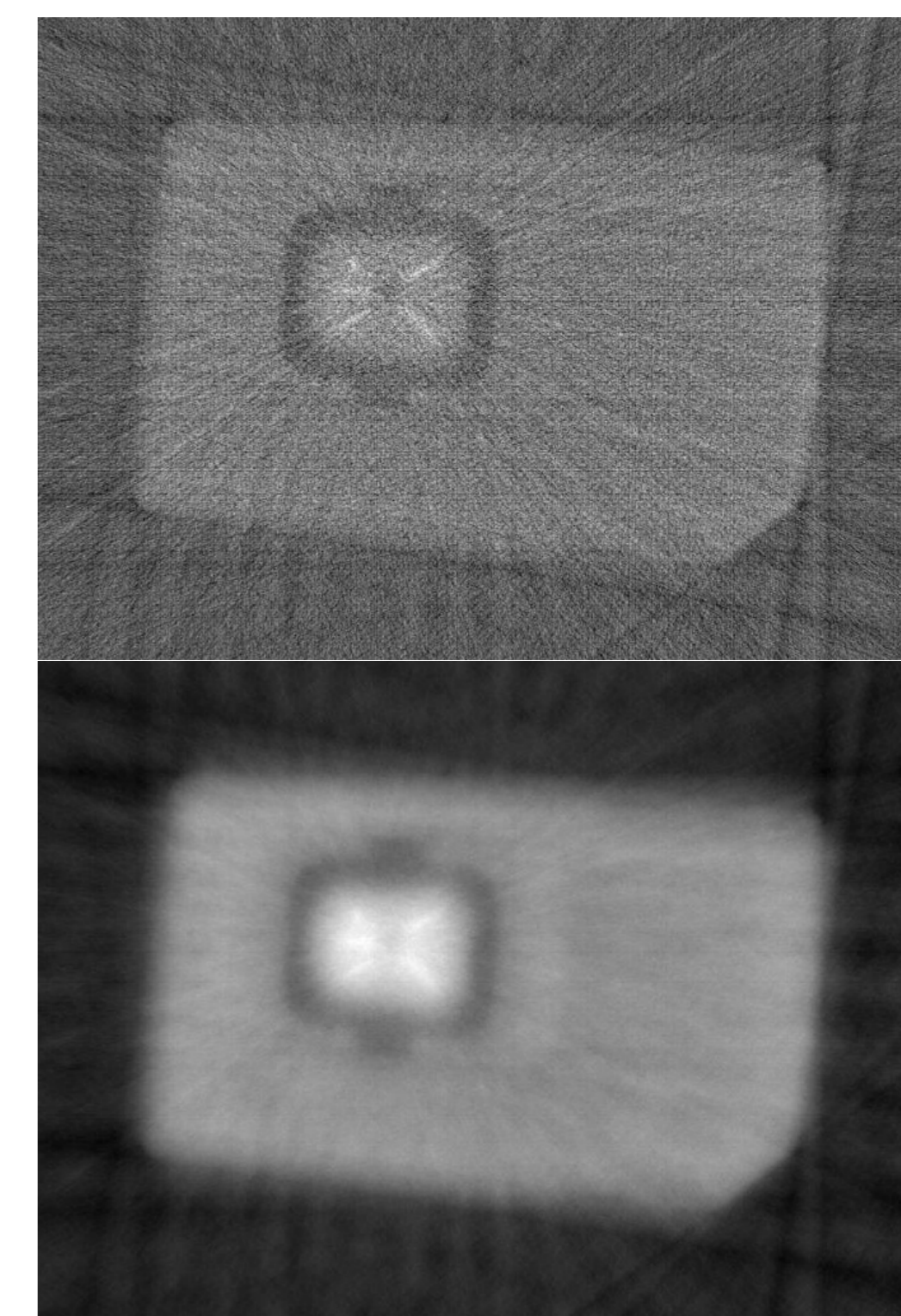

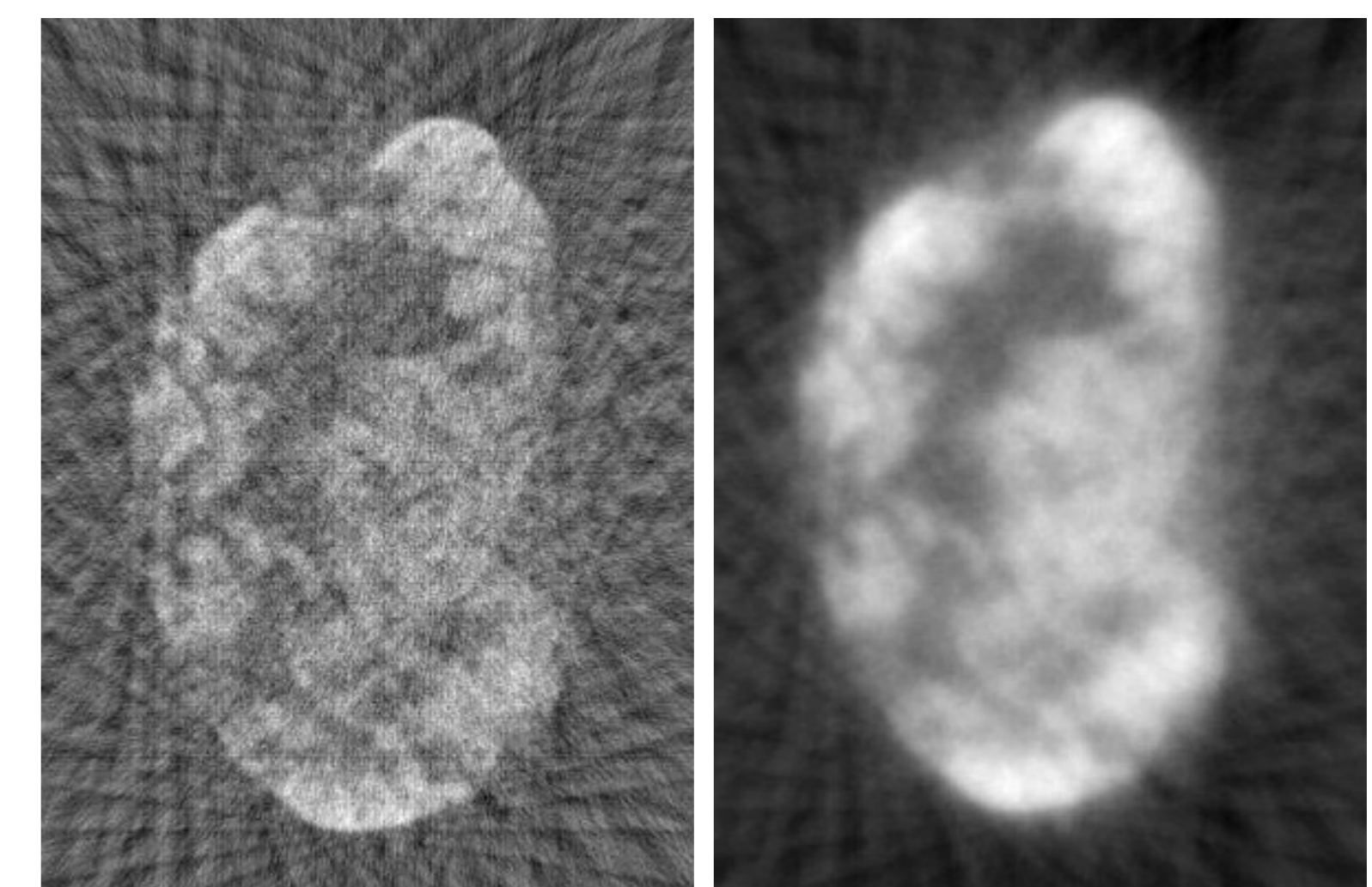**Figure 4:** CT image of a SIM-card with filtered back projection above and Tikhonov regularization below.

Figure 5: CT image of a limestone with filtered back projection on left and Tikhonov regularization on right.

Discussion

Both methods give quite reasonable results for our objects. Images of SIM-cards are precise enough to show even the wires in SIM-card plate and small holes in the limestone. Beam hardening is detected in all pictures, since the wires in the SIM-card are broadened and the outer parts of the stone are clearly brighter than the centrum. The effects are not, however, too strong to ruin the reconstructions.

The reconstructions with Tikhonov and back projection methods differ remarkably, since filtered back projection gives sharper edges, but poorer contrast. Tikhonov regularization provides some smoothing, so it's natural that the images are a bit blurred. Tikhonov method on the other hand has much better constrast, so different materials are more easily detected from the image.