Inverse problems course Exercise 5 (February 17–19, 2015)
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Related book sections (Mueller & Siltanen 2012): 5.2, 5.3 and 5.4.2.

Theoretical exercises:

T1. Use the variational method of then book's section 5.2 to show that the minimum-norm solution of the equation $Bz = \mathbf{b}$ satisfies the normal equations

$$B^T B z_0 = B^T \mathbf{b}.$$

T2. Write the following generalized Tikhonov regularized solution in stacked form:

$$\arg\min_{z \in \mathbb{R}^n} \{\|Az - m\|^2 + \delta \|L(z - z^*)\|^2\}.$$

Here the fixed vector $z^* \in \mathbb{R}^n$ is assumed to be known *a priori*.

Matlab exercises:

M1. Consider the one-dimensional deconvolution problem. Use the L-curve method (Section 5.4.2 of the book) to the classical Tikhonov regularization ($L = I$ in the penalty term) to find a good regularization parameter. Plot the reconstruction you get from the choice of regularization parameter and compute the relative error.

M2. Repeat exercise M1 with the regularization matrix L replaced by derivative penalty, i.e. the rows of the matrix L should have the form

$$0, \ldots, 0, -1, 1, 0, \ldots, 0.$$

Find the best regularization parameter according to the L-curve method. Is the reconstruction less or more oscillatory than in M1?

M3. Use the stacked form method to compute Tikhonov regularized reconstructions for the tomography problem. Try to take the resolution parameter N and the number T of projection angles as big as possible while keeping the computation time in less than 30 seconds and while not using all the computer memory. Can you compute the SVD of the matrix A at that resolution, or is it too much for your computer?