Course announcement

ERGODIC THEORY

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- Advanced studies in mathematics, 5 credit points
- Period IV / spring 2015 (9.3.–3.5. excluding Easter break 2.4.–8.4.)
- Weekly schedule: 2×2 hours of lectures + exercises according to mutual agreement

  Urgent! If you plan to take the course, please answer the poll concerning the scheduling of the lectures no later than **Wednesday 18.2.** A link to the poll is on the website below.
- Website: Department homepage → Studies → Courses spring 2015 → Ergodic theory

What is ergodic theory? Ergodic theory originated in physics, in an attempt to derive thermodynamics from the Hamiltonian mechanics of particle systems. In modern terms, the deterministic time-evolution of such a system can be described by group of transformations $T_t : X \to X$ of a measure space $(X, \mathcal{B}, \mathrm{m})$, where the measure m is "invariant". (Namely, the Hamiltonian flow preserves the phase space volume.) Ergodic theory can be viewed as the abstract study of deterministic systems described by transformations of measure spaces. What is meant by a "deterministic system" here is left intentionally vague; ergodic theory has found applications not only in physics but in other branches of mathematics and science, including probability theory, number theory, information theory, biology and ecology.

Objective. The course aims to equip the student with a solid understanding of the elements of ergodic theory that are useful in various branches of mathematics. In particular, it will serve as a springboard for further studies in the theory of dynamical systems.

Topics covered. We will study the ergodic theory of deterministic processes modeled by abstract transformations of measurable spaces and dwell on the special case of continuous transformations of compact metric spaces. In particular, we will prove the recurrence theorems of Poincaré and Kac, as well as the ergodic theorems of von Neumann and Birkhoff. Depending on the interest of the audience, and time permitting, we will also touch on the ergodic theory of random processes. We will then move on to the topic of measure-theoretic entropy, and see how it ties in with the notion of information and with the problem of classifying transformations of measure spaces.

Prerequisites. The student should know the basics of measure theory, Lebesgue integration and elementary topology, and be comfortable with the notions of Banach/Hilbert spaces (in particular L^p spaces) and bounded linear operators on them. We will use several theorems (Hahn–Banach, Riesz representation, Radon–Nikodym, ...) from real and functional analysis, which will be recalled during the lectures; the courageous student could take the course without prior knowledge of these results, but understanding the precise statements is essential, for which the mentioned prerequisites are key.