Introduction to LaTeX
Exercise Sheet 2 (Group 5)

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The `.tex` file containing your solutions to this exercise sheet should be emailed to `clifford.gilmore@helsinki.fi` before 15:00 on 20th March. The subject line of the email should be *Latex Exercises 2* and the file name should be of the form `SurnameExercise2.tex`, e.g `GilmoreExercise2.tex`.

The produced document should contain enough text to fill two pages. If you can't think of anything to write then you can find random text from Lorem Ipsum at http://www.lipsum.com/

Note, in the questions where you define a `newtheorem` environment, the theorem number will be automatically generated by LaTeX. So it's okay if the theorem's number is different in your solution!

1. Create a document titled, *LaTeX Solutions 2*, with you as the author.
2. Add the following to your document:
   (a) A section titled *Miscellaneous Mathematics*.
   (b) Inside the section add a subsection titled *Text Style Maths*.
   (c) Inside this subsection include the following text, where the mathematics appears inline with the text:
      (i) Pythagoras states for a right angled triangle with side lengths a, b, c, then $a^2 + b^2 = c^2$.
      (ii) Euler's identity states that $e^{i\pi} + 1 = 0$.
3. Add a new subsection to your document with the title *Display Style Maths*. Inside this subsection include the following, where the mathematics appears in the `equation` environment
   (a)
$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k^2} = \frac{\pi^2}{6} \tag{1}$$

(b) Pascal's rule is

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1} \tag{2}$$

(hint: you will need the `amsmath` package and the `binom` command here)

(c)

$$\int_0^{\frac{\pi}{2}} \cos x \mathrm{d}x$$

(d) For a separable Banach space X, a bounded linear operator T is hypercyclic if there exists a vector $x \in X$ such that its orbit under T is dense in X, i.e.

$$\overline{\{T^n x : n \geq 0\}} = X.$$

(hint: you need the `overline` command here)

4. Add a new subsection, titled *Aligning Equations*, and using the `align` environment include the following equations:

$$\begin{aligned}\sum_{j=k+1}^{\infty} 2^{n_k - n_j} \|y^{(j)}\| &\leq \sum_{j=k+1}^{\infty} 2^{-j} \\ &= 2^{-k-1} + 2^{-k-2} + \cdots \\ &= 2^{-k}\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\right) \\ &= 2^{-k} \sum_{i=1}^{\infty} \left(\frac{1}{2}\right)^i = 2^{-k}.\end{aligned}$$

(hint: you need the \`lVert` and \`rVert` commands here)

5. Create a new section titled, *Linear Chaos*.

Define your own Theorem environment with the `newtheorem` command and then use it to state the below theorem.

Theorem 2.1. Let T be a bounded linear operator on a separable Banach space X. If the spans of the below subspaces are dense in X, then T is hypercyclic:

$$\begin{aligned}X_0 &= \{x \in X : Tx = \lambda x \text{ for some } \lambda \in \mathbb{K}, |\lambda| < 1\}, \\ Y_0 &= \{x \in X : Tx = \lambda x \text{ for some } \lambda \in \mathbb{K}, |\lambda| > 1\}.\end{aligned}$$

6. Using the `proof` environment accessed through the `amsthm` package, add a proof for the above theorem. (You don't need to give the real proof, any paragraph of text will do)

7. Create a *Lemma* environment using the `newtheorem` command. The numbering of the lemma should be in the same sequence as the above Theorem. Add the following lemma.

   Lemma 2.2. Let X be a separable Banach space. If $x \in X$ is a hypercyclic vector for the bounded linear operator $T\colon X \to X$, then each $T^n x$ is also a hypercyclic vector for T.