

Department of Mathematics and Statistics, University of Helsinki
 Numerical methods and the C language, fall 2010

Workshop 12

Mon 13.12. at 16–18 B322

1. By using the program `odeint` included in e.g. `myod1.c` on the www-page, solve the second-order constant coefficient differential equation $y'' + ay' + by = g(x)$ initial value problem $y(0) = 0$, $y'(0) = 1$ when $a = 2$, $b = 3$, $g(x) = \exp(-x)$ and compare to the exact solution $0.5 * r + r * (c_1 * \cos(x/c_2) + (c_2) * \sin(x/c_2))$, $r = \exp(-x)$, $c_1 = -0.5$, $c_2 = \text{pow}(2.0, -0.5)$. *Hint:* The situation can be reduced to solving the system of equations

$$\begin{cases} \frac{dy_1}{dx} = y_2(x), \\ \frac{dy_2}{dx} = g(x) - ay_2(x) - by_1(x) \end{cases}$$

2. The so called *Lorentz system of differential equations* occur in e.g. weather forecasting:

$$\begin{aligned} \frac{dy_1}{dx} &= s(y_2(x) - y_1(x)), \\ \frac{dy_2}{dx} &= ry_1(x) - y_2(x) - y_1(x)y_3(x), \\ \frac{dy_3}{dx} &= y_1(x)y_2(x) - by_3(x). \end{aligned}$$

By using `rkdumb` or `odeint`, solve this in the interval $[0, 8]$ when $s = 10$, $r = 128$, $b = 8$ and the initial values are $7.7, -15.6, 90.4$.

3. Modify the program `myod1.cpp` as to draw pictures of the solution function of the following initial value problem. The hypergeometric function $F(a, b; c; x)$ satisfies

$$\begin{aligned} \frac{dF(a, b; c; x)}{dx} &= \frac{ab}{c} F(a+1, b+1; c+1; x) \quad \text{and} \\ F(a, b; c; 0) &= 1. \end{aligned}$$

For $x \in (0, 1)$ and $a, b \in (0, 1)$

$$\begin{aligned} \frac{dF(a-1, b; c; x)}{dx} &= \frac{(a-1)}{x} [F(a, b; c; x) - F(a-1, b; c; x)], \\ \frac{dF(a, b; c; x)}{dx} &= \frac{(c-a)F(a-1, b; c; x) + (a-c+bx)F(a, b; c; x)}{x(1-x)}. \end{aligned}$$

Hence the IVP

$$\begin{aligned} \frac{dy_1}{dx} &= \frac{a-1}{x} [y_2(x) - y_1(x)], \\ \frac{dy_2}{dx} &= \frac{(c-a)y_1(x) + (a-c+bx)y_2(x)}{x(1-x)}, \\ y_1(x_1) &= F(a-1, b; c; x_1), \\ y_2(x_1) &= F(a, b; c; x_1) \end{aligned}$$

has solution $y_1 = F(a - 1, b; c; x)$, $y_2 = F(a, b; c; x)$. Set concrete values such as $x_1 = 0.1$, $a = 1.2$, $b = 2.3$, $c = 3.5$ and compute the numerical and the exact solution on $[0.1, 0.9]$. Also compute the difference between these.

4. Use the program `odeint.c` to compute the integral

$$\int_0^x 10 \sin(2^n \pi t) dt, n = 1, 2, \dots, x \in (0, 1),$$

and print the difference to the exact value.

5. We consider for $h > 0$ a quadrature formula of the form

$$\int_0^{3h} f(x) dx = w_1 f(0) + w_2 f(2h) + w_3 f(3h)$$

where the weights $w_j, j = 1, 2, 3$, are chosen so that this holds as equality for each of the three functions $1, x, x^2$.

- (a) Find the coefficients $w_j, j = 1, 2, 3$.
- (b) Show that the formula also holds as equality for $f(x) = ax^2 + bx + c$.
- (c) What is the error if $h = 1, f(x) = 1 + x + x^2 + x^3$?