Department of Mathematics and Statistics, University of Helsinki
Numerical methods and the C language, fall 2010

Workshop 7

Mon 1.11. at 16–18 B322

1. Compute the value of the integral $I_m = \int_0^1 \cos(\pi m x)dx$ with four different methods (a) The trapez formula, (b) Romberg integration (`NR::qromb` or in GSL the method `romberg` at the www-page), (c) Gauss (NRC) or Gauss-Kronrod (GSL) method, (d) a method of your choice (but different from (a)-(c)). Compare the accuracy of the results.

2. On the www-page you will find a data file of xy-pairs. Find the value of σ that minimizes

$$g(\sigma) \equiv \sum_{k=0}^{m-1} (y[k] - f(\sigma, x[k]))^2,$$

   where $f(\sigma, x) = (x/\sigma) * \exp(-x^2/(2.0 * \sigma^2))$.

   Hint: Compute the value of $g(\sigma)$ when $\sigma = 0.1 : 0.1 : 10$ and choose the one that minimizes $g(\sigma)$.

3. Modify the program `mypolyarea.cpp` as to calculate the area of:

   a) A polygonal domain with the shape of the letter C:

```
x  0 6 6 4 4 2 2 4 4 6 6 0 0
y  0 0 3 3 2 2 5 5 4 4 7 7 0
```

   b) A polygonal domain with the shape of two triangles, where the polygonal line intersects itself:

```
x  0  1 1 -1 -1 0
y  0 -1 1 -1  1 0
```

4. Let A, B be $m \times n$ and $p \times q$ matrices. Their Kronecker product $\mathrm{Kron}(A, B)$ (also called tensor or direct product) is the $mp \times nq$ matrix

```
(a(1,1)B ... a(1,n)B; ...; a(m,1)B ... a(m,n)B );
```

   For simplicity we write here $\mathrm{Kron}(A, B) = [A, B]$. The program Kronecker on the www-page gives an implementation of this product. Verify experimentally the following properties

   (a) $[A, [B, C]] = [[A, B], C]$,

   (b) $[A, B + C] = [A, B] + [A, C]$,

   (c) $[A^{-1}, B^{-1}] = [A, B]^{-1}$

   (d) $[A^T, B^T] = [A, B]^T$

   (e) If B and C have SVDs $B = U_1 W_1 V_1^T$ $C = U_2 W_2 V_2^T$, then $[B, C] = [U_1, U_2][W_1, W_2]([V_1, V_2]^T)$.

5. On the course www-page some data of x, y, z values are given. Find the coefficients a, b, c, d that for the model $ax + by + c = z$ provides the best fit to this data.

```
20 3
  -9.8453630878801288e+00   9.0916121374310990e+00   9.1754800449942611e+00
   9.7116816042511172e+00  -9.0877979244514364e+00   1.9372308402961265e+00
   4.3915465727409098e+00   5.2584261238847052e+00   7.5309994339621626e+00
   4.5212450132338544e+00   8.1883741813657682e+00  -8.2489441792708558e+00
   7.3507759335221614e+00   7.1167275389268658e+00   3.8727680751461389e+00
  -5.4234284048077779e+00  -7.5549827877222482e+00   6.1411724407883233e+00
  -8.4860762015385909e+00   7.1556082168387292e+00  -4.3861598588461801e+00
   5.1308149821734128e+00   2.8066688463122906e+00  -1.3337349991005543e-01
   1.8982809744301630e+00  -9.5789346422902000e-01  -8.9231700072638542e+00
  -4.5691388680455927e-02   3.0140608330322713e+00  -2.6558042283429786e+00
  -1.7275130523962494e+00  -5.1248048036940421e+00  -8.0956313750220605e+00
  -6.9364306223282730e+00  -6.3445755542929172e+00  -7.2347443817345169e+00
  -9.2278480246792771e+00   6.6380236934116219e+00  -8.4944038272343594e+00
   9.0830409056893746e+00   7.6467686135539639e+00   2.8016523005448524e-01
   4.8023768210794673e+00   5.8940053712083049e+00  -6.9013514355296977e+00
  -1.2993257824794044e+00   2.9652750645602470e+00  -4.7813563117670626e+00
  -7.9926183996687730e+00  -3.7045263190309172e+00   2.2089177613188129e+00
   9.3095345140013546e+00   9.5531135422890596e+00   6.3075453957112249e+00
  -5.5716912707182074e+00   8.2119140206891643e+00   3.7575342244270882e-01
  -8.1326615429169777e+00   3.1410090407081923e+00  -2.0937765911658186e+00
```