Bayesian inversion for 3D dental X-ray imaging

Samuli Siltanen
Department of Mathematics and Statistics
University of Helsinki

China-Finland-Germany workshop on inverse problems
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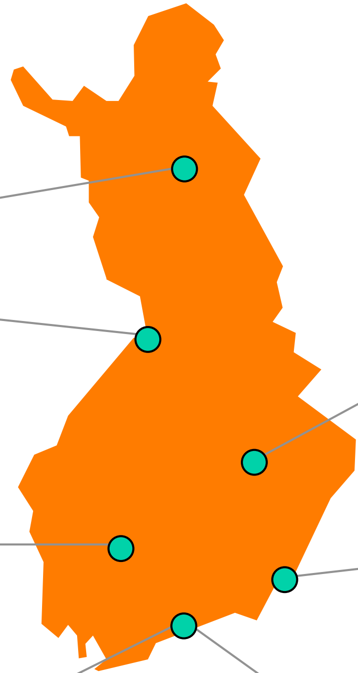
Finnish Centre of Excellence in Inverse Problems Research



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Research institute of mathematics, computer science and statistics

http://math.tkk.fi/inverse-coe/

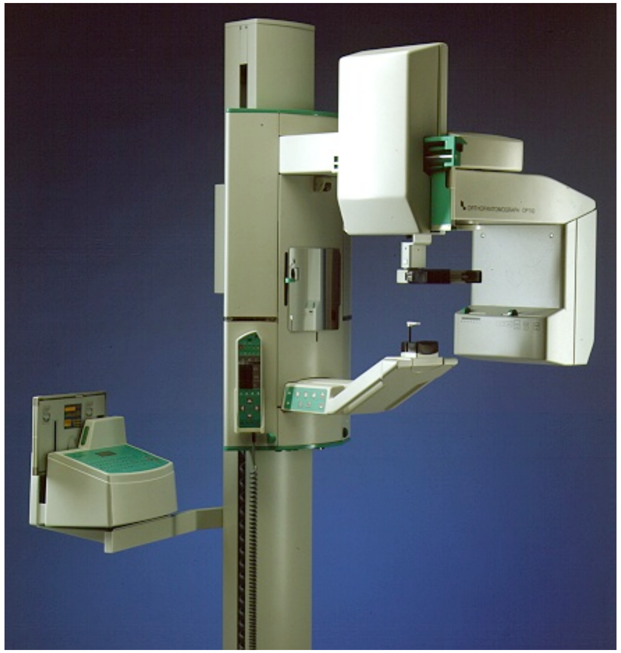
A series of projects started in 2001 aiming for a new type of low-dose 3D imaging

The goal was a mathematical algorithm with

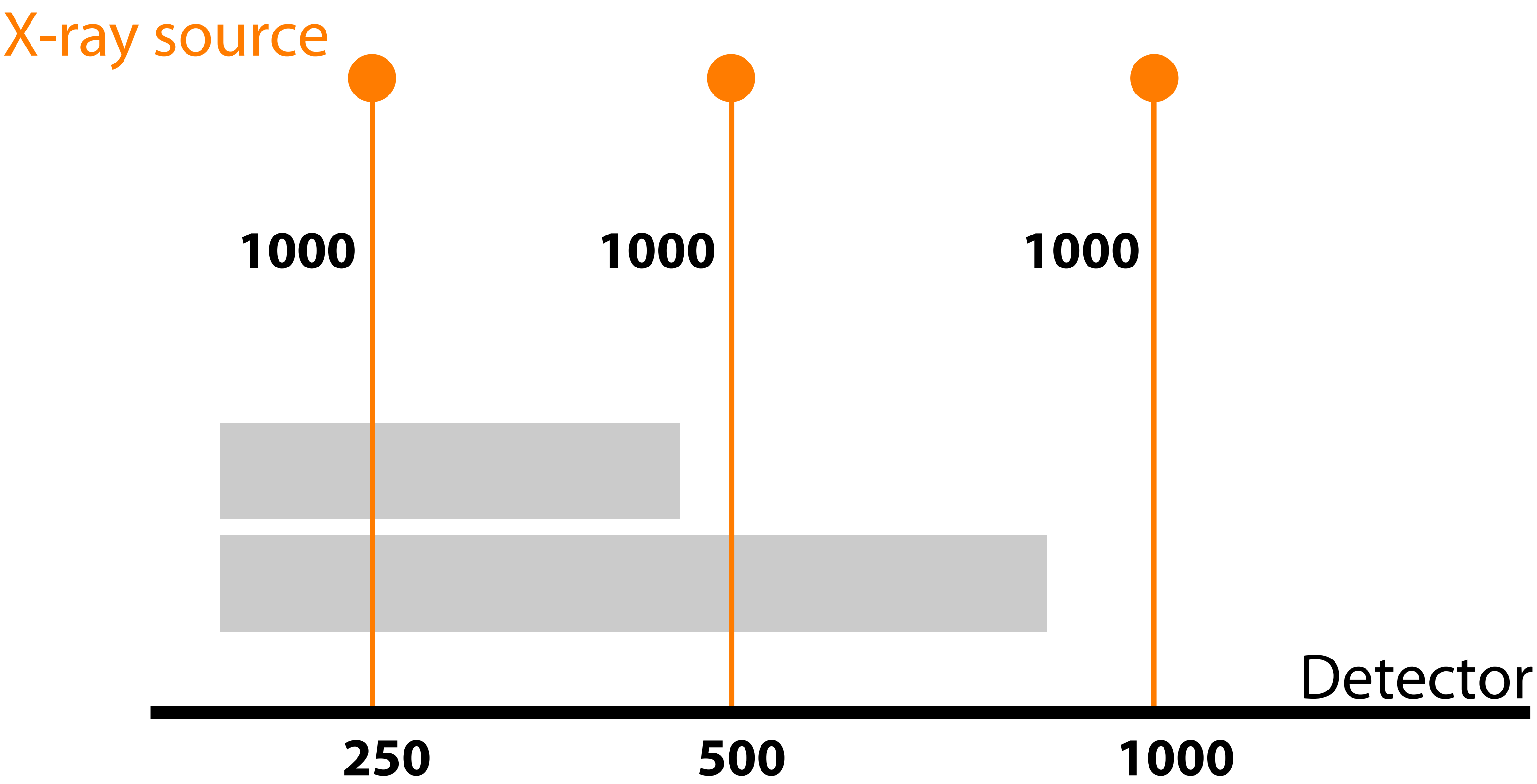
Input: small number of digital X-ray images taken with any X-ray device

Output: three-dimensional reconstruction with quality good enough for the clinical task at hand

Products of GE Healthcare in 2001:

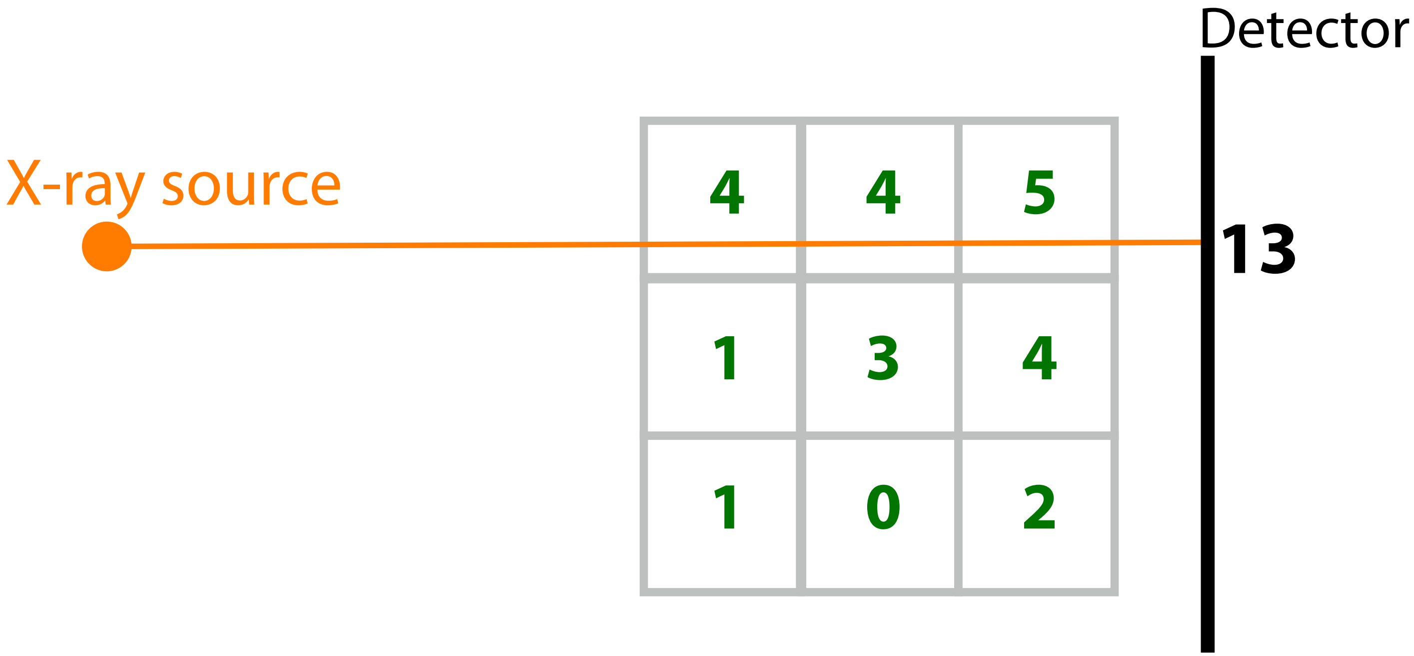


X-ray images as measurements

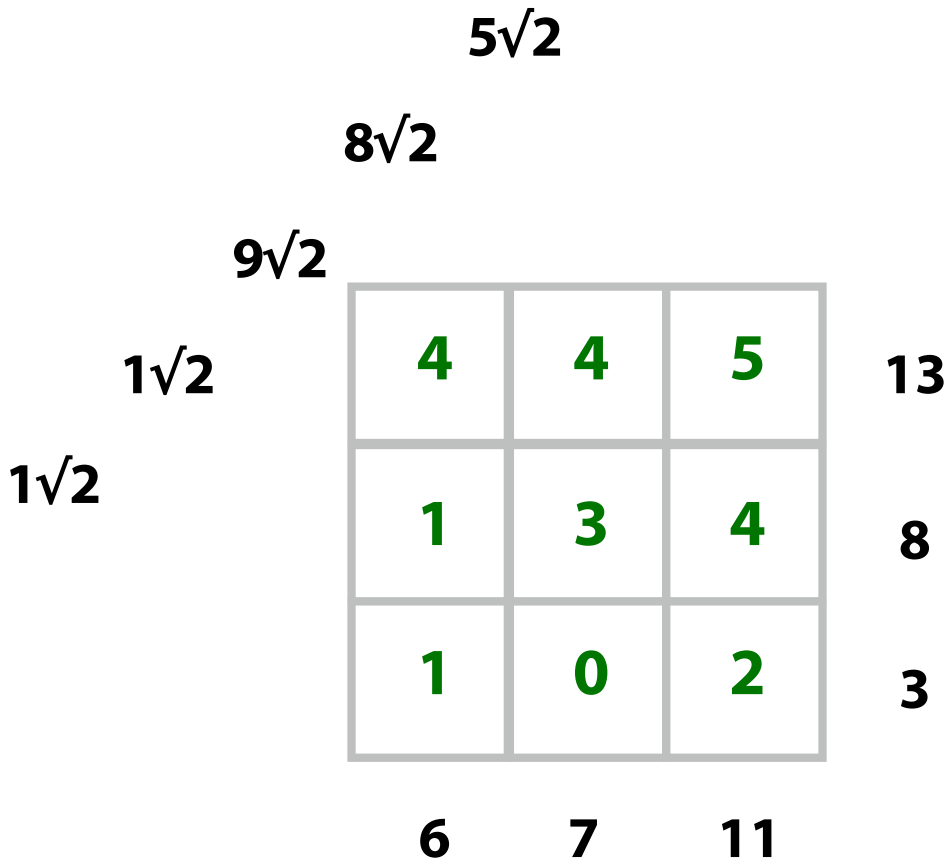


Logarithm	**5,5**	**6,2**	**6,9**
Density	**1,4**	**0,7**	**0,0**

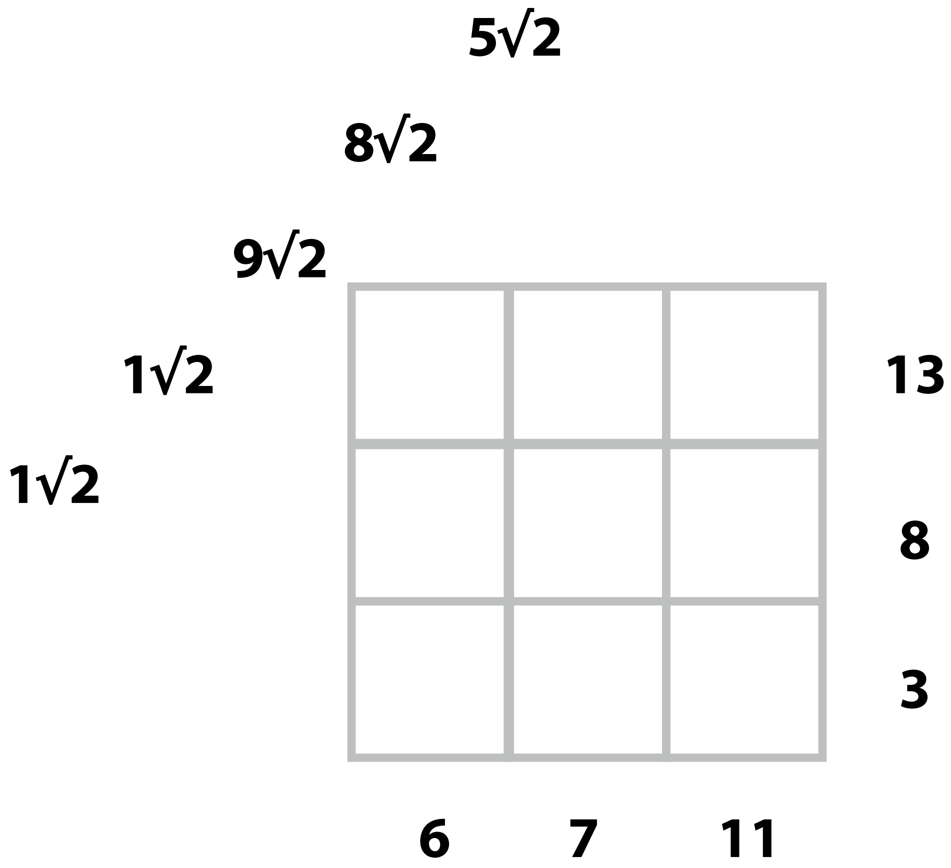
Every X-ray measures the sum of attenuation through tissue



Direct problem of tomography is to find the radiographs from given tissue

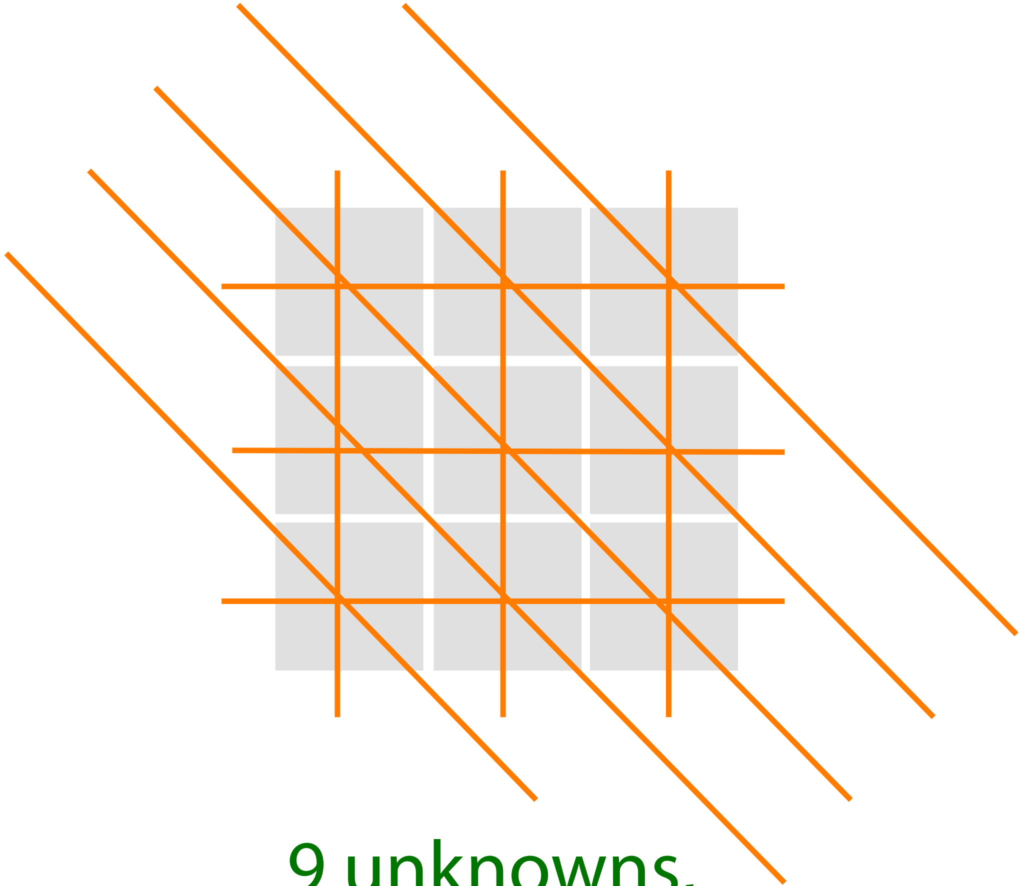


Inverse problem of tomography is to find the tissue from radiographs

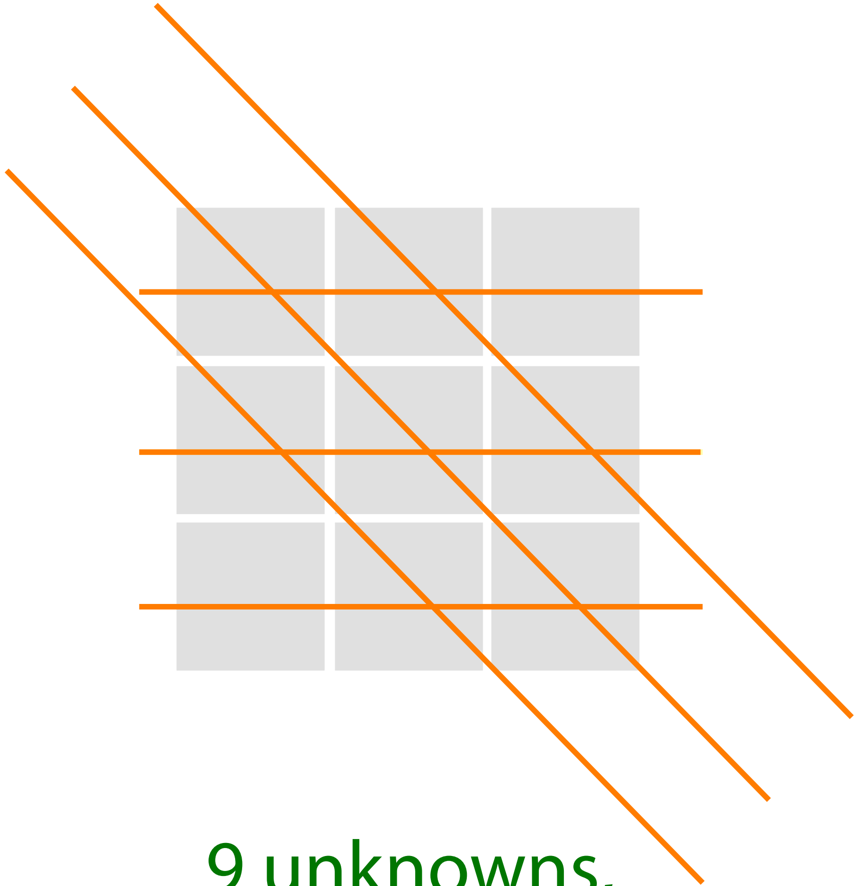


9 unknowns, 11 linear equations

The limited angle problem is harder than the full angle problem



9 unknowns,
11 linear equations



9 unknowns,
6 linear equations

In limited angle 3D imaging there are many tissues matching the radiographs

8√2

9√2

1√2

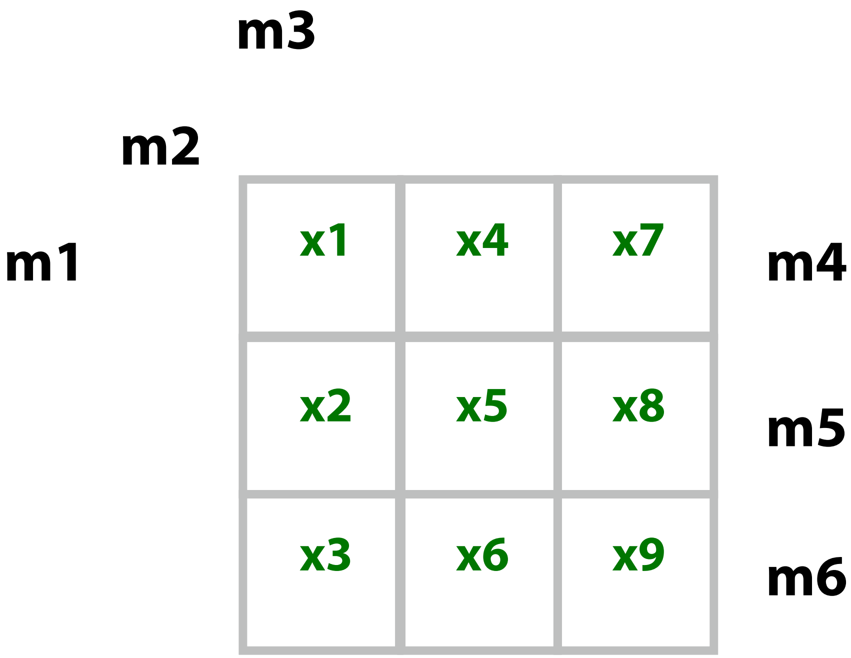
4	4	5	13
1	3	4	8
1	0	2	3

5	6	2
1	5	2
4	0	-1

9	1	3
1	0	7
3	0	0

a priori information is needed!

We write the reconstruction problem in matrix form and assume Gaussian noise



$$x = [x_1, x_2, \dots, x_9]^T$$
$$m = [m_1, m_2, \dots, m_6]^T$$

$$Ax = m$$

Measurement m=Ax+e with Gaussian noise ε of standard deviation σ leads to the following likelihood distribution:

$$p(m|x) = p_\varepsilon(Ax - m) \sim \exp(-\frac{1}{2\sigma^2}\|Ax - m\|_2^2)$$

Bayes formula combines measured data and *a priori* information together

We reconstruct the most probable 3D tissue in light of

1. Available radiographs and
2. Physiological *a priori* information

Bayes formula gives the *posterior distribution* p(x|m):

$$p(x|m) \sim p(x)\, p(m|x)$$

Prior distribution, or tissue model — $p(x)$

Likelihood distribution, or measurement model — $p(m|x)$

We recover x as a point estimate from p(x|m)

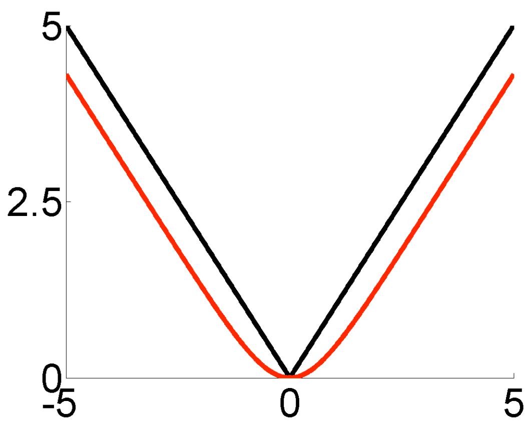
We build a prior distribution for dental tissue using total variation prior

Positivity constraint:

$$p_+(x) = \begin{cases} 0 & \text{if } x_j < 0 \text{ for some } j \\ 1 & \text{otherwise} \end{cases}$$

Approximate total variation penalty:

$$p_{\mathrm{TV}}(x) = \exp\left(\alpha \sum_{\text{neighbors}} |x_\ell - x_k|_\beta\right)$$



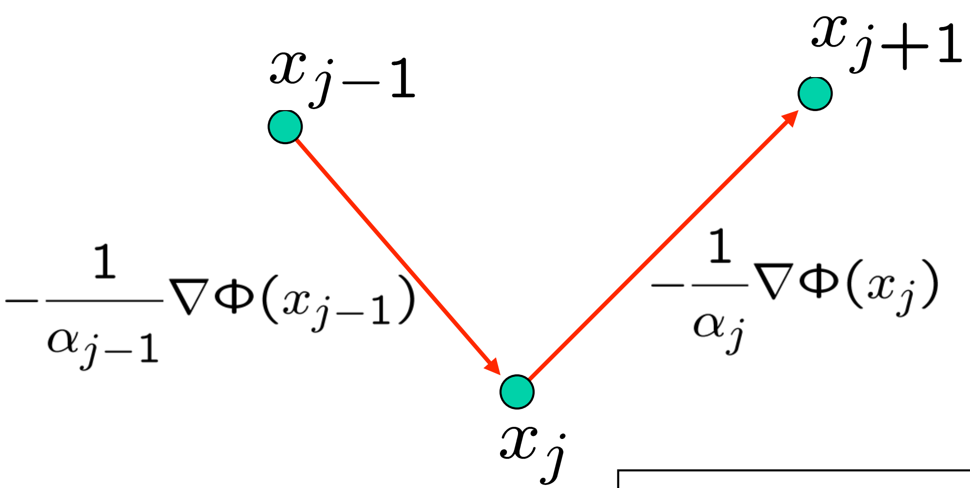
$$|t|_\beta = \frac{1}{\beta} \log(\cosh(\beta t))$$

Computation of the MAP estimate

Large scale optimization problem:

$$x_{\text{MAP}} = \arg\min_{x_j \geq 0} \{\frac{1}{2\sigma^2}\|Ax - m\|_2^2 + \alpha \sum_{\text{neighbors}} |x_\ell - x_k|_\beta\}$$

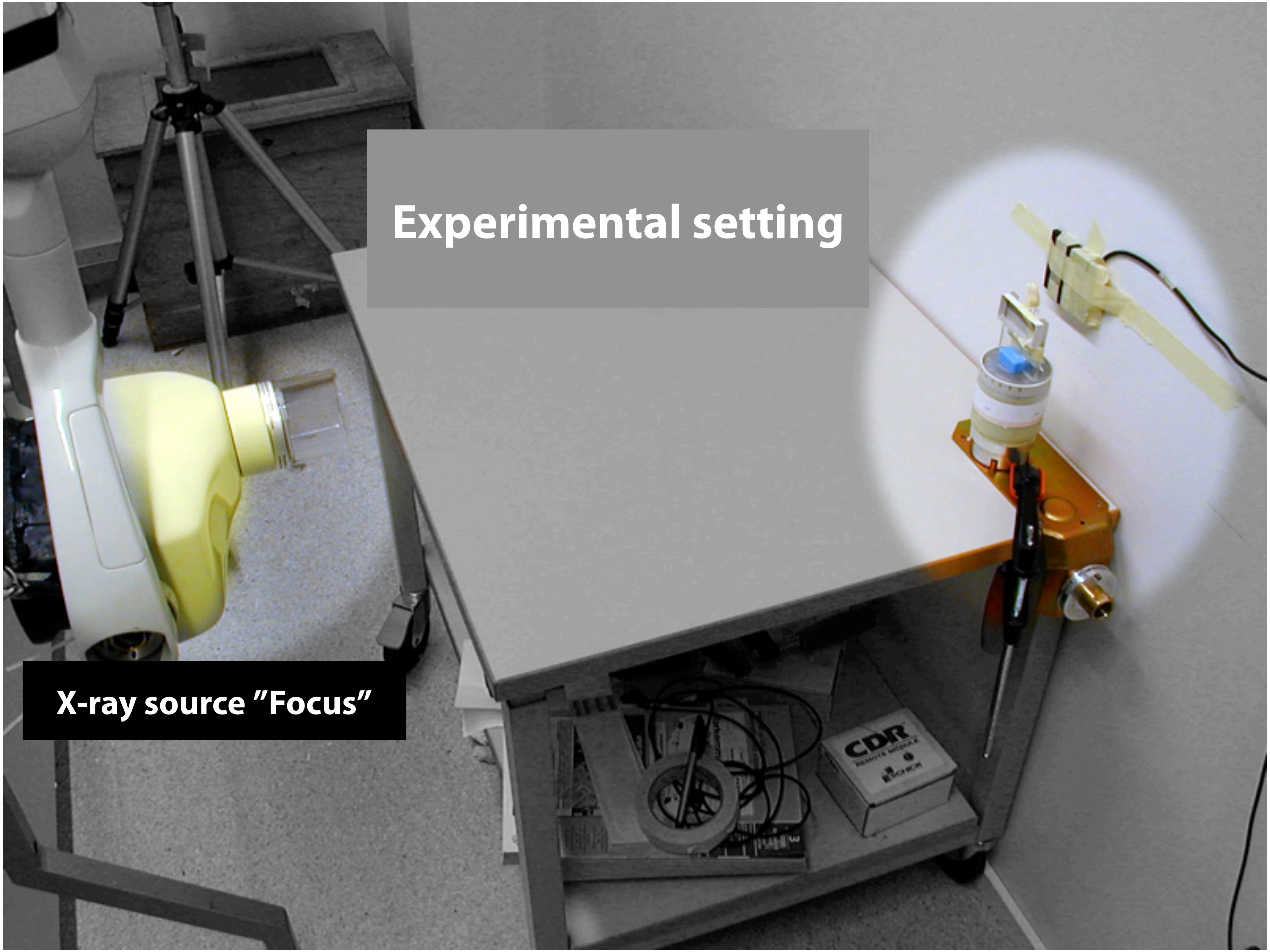
We use the gradient method of Barzilai & Borwein, which is a modification of Euler's steepest descent method.

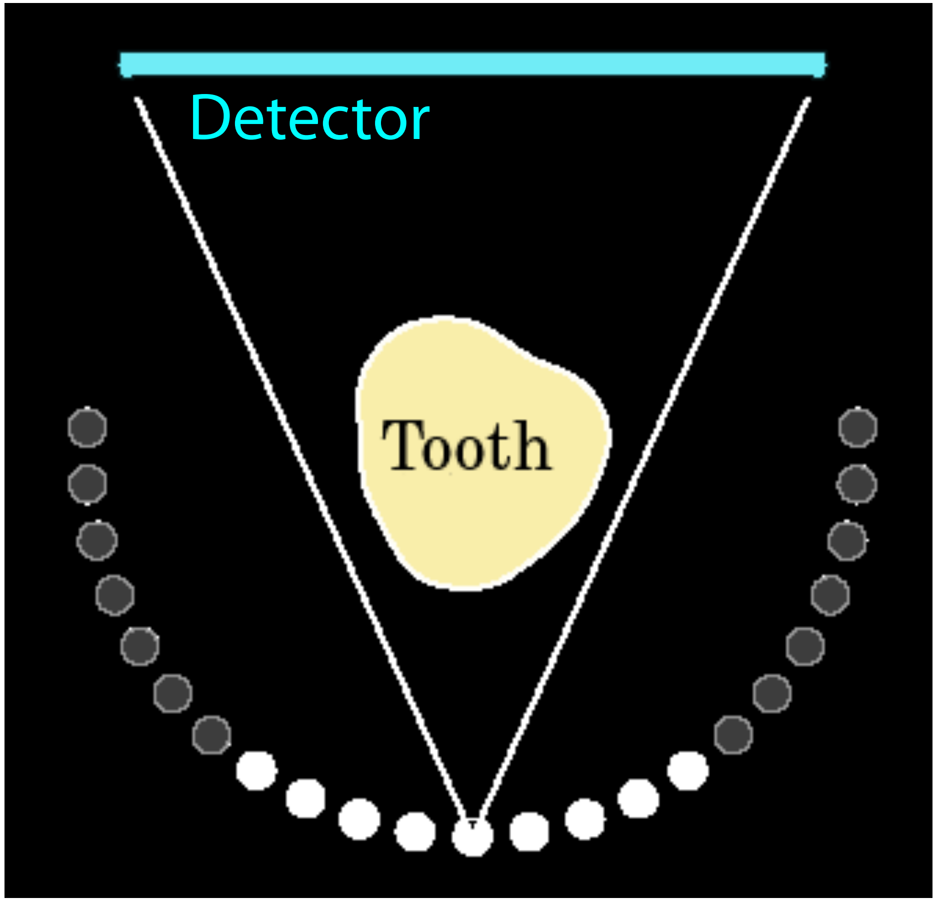


Step size differs from Euler's:

$$\alpha_j = \frac{(x_j - x_{j-1}) \cdot (\nabla\Phi(x_j) - \nabla\Phi(x_{j-1}))}{(x_j - x_{j-1}) \cdot (x_j - x_{j-1})}$$

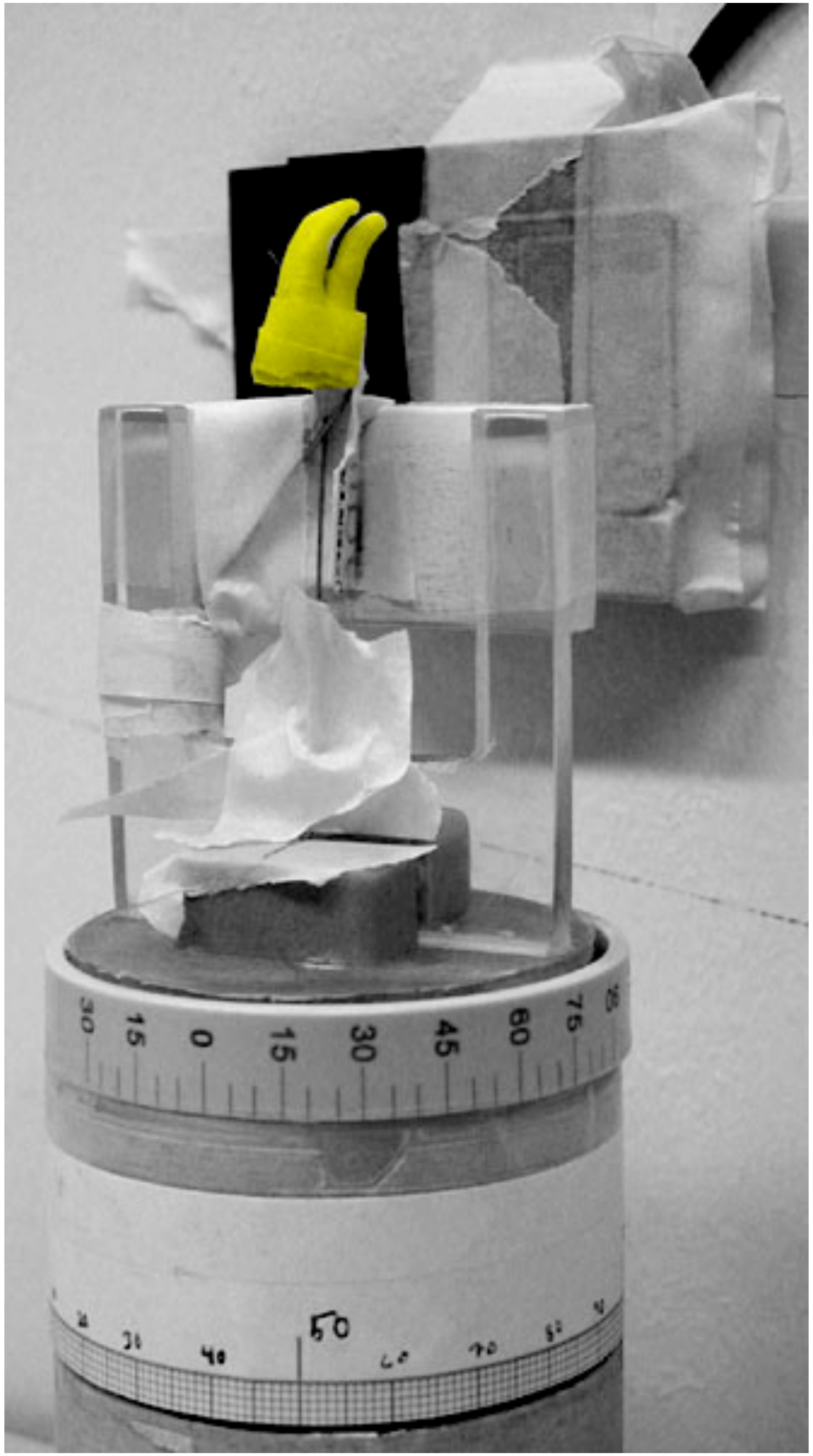
Experimental setting



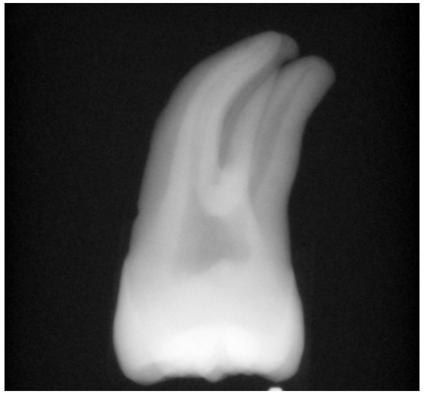


X-ray source positions

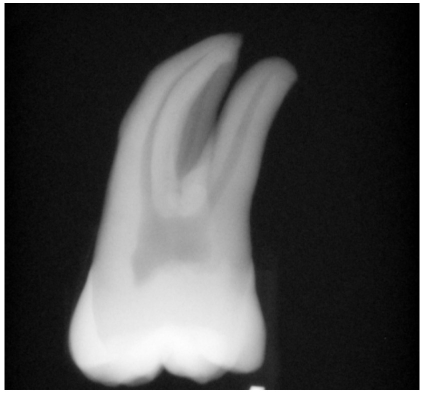
Tooth donated to science
by Helena Sarlin. Thanks!



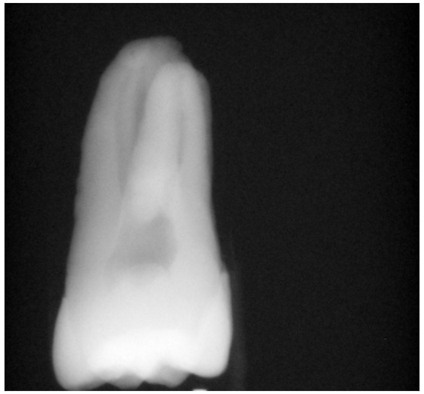
The projection images look like this



0



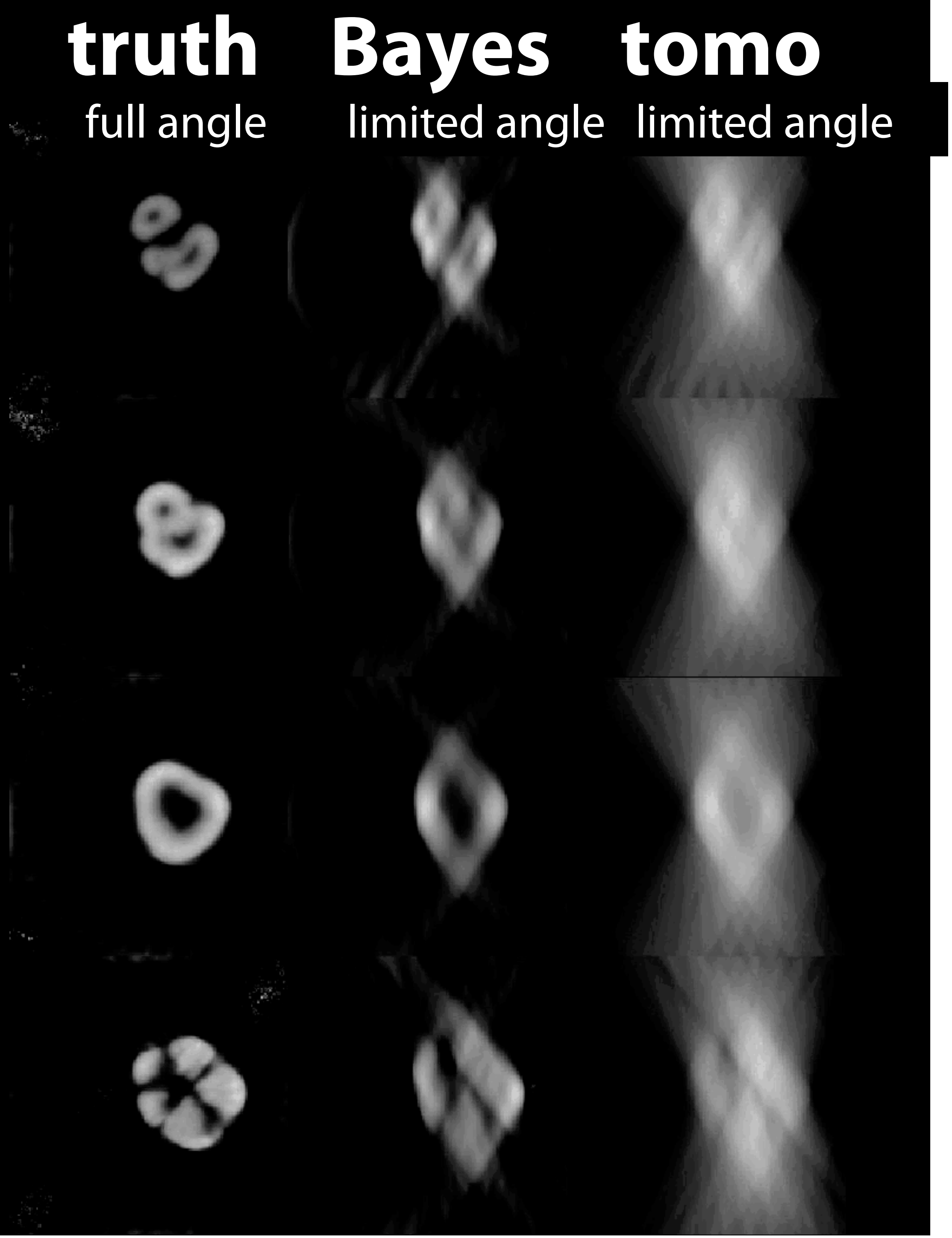
30



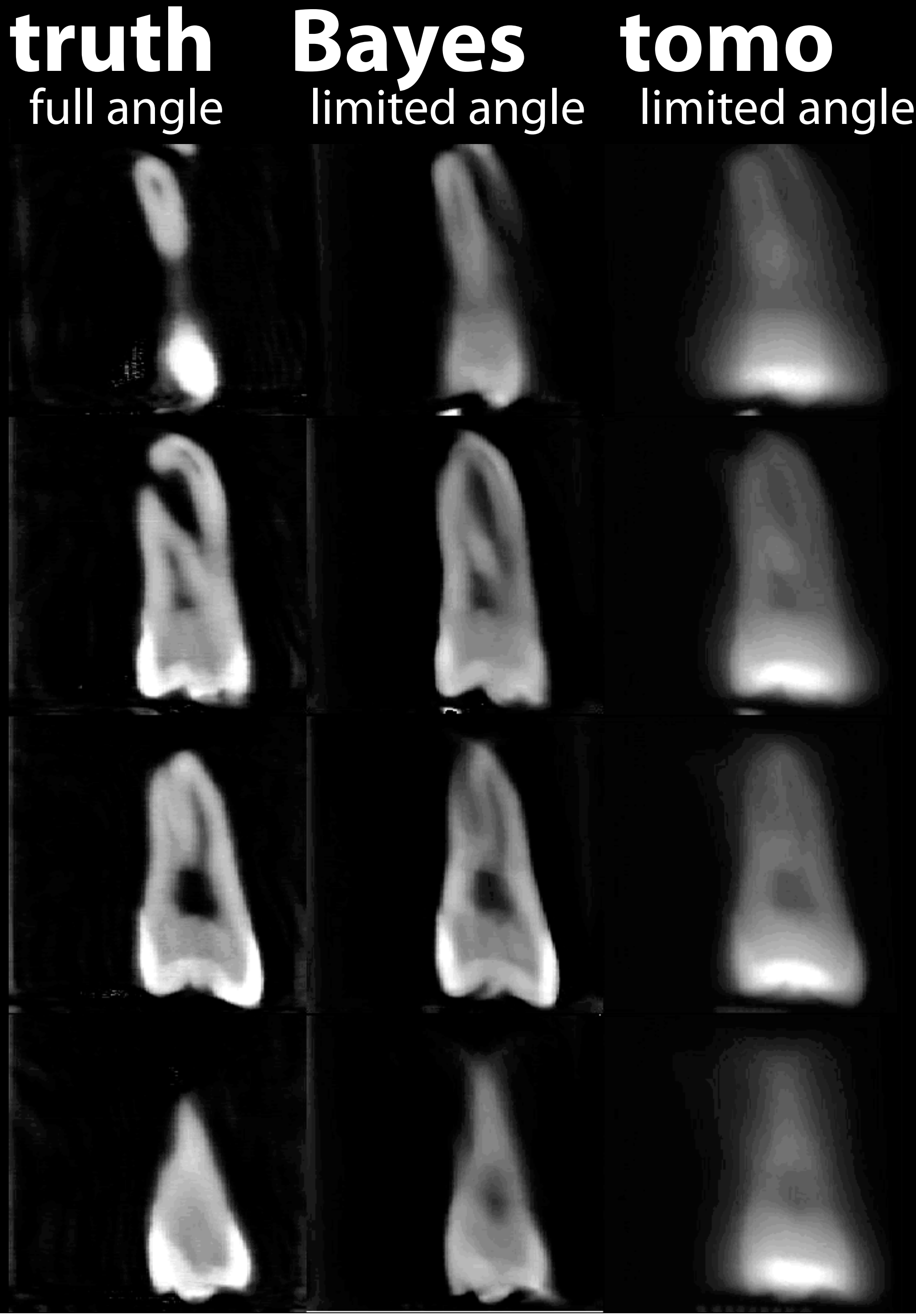
60

90

Horizontal slices:

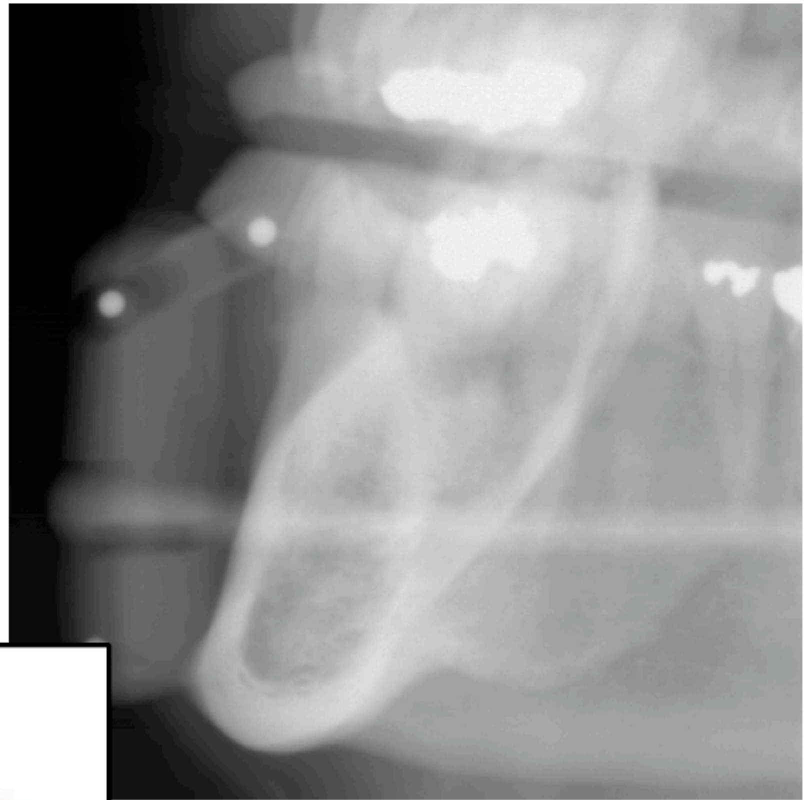


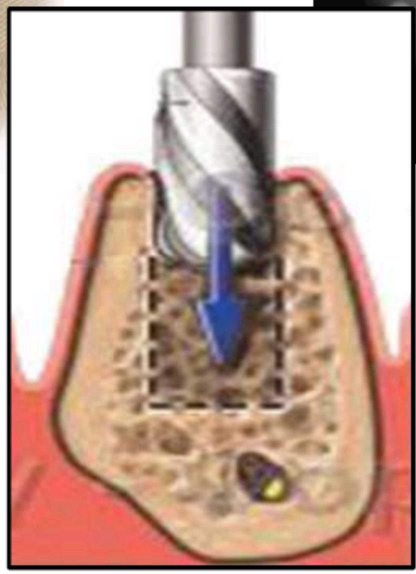
Vertical slices:



Kolehmainen, S, Järvenpää, Kaipio, Koistinen, Lassas, Pirttilä, Somersalo (2003)

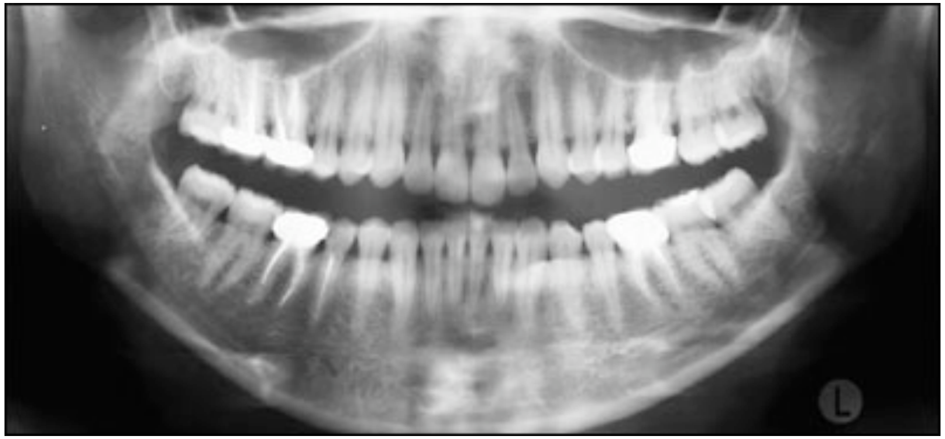
2D projection radiograph is not enough for dental implant planning





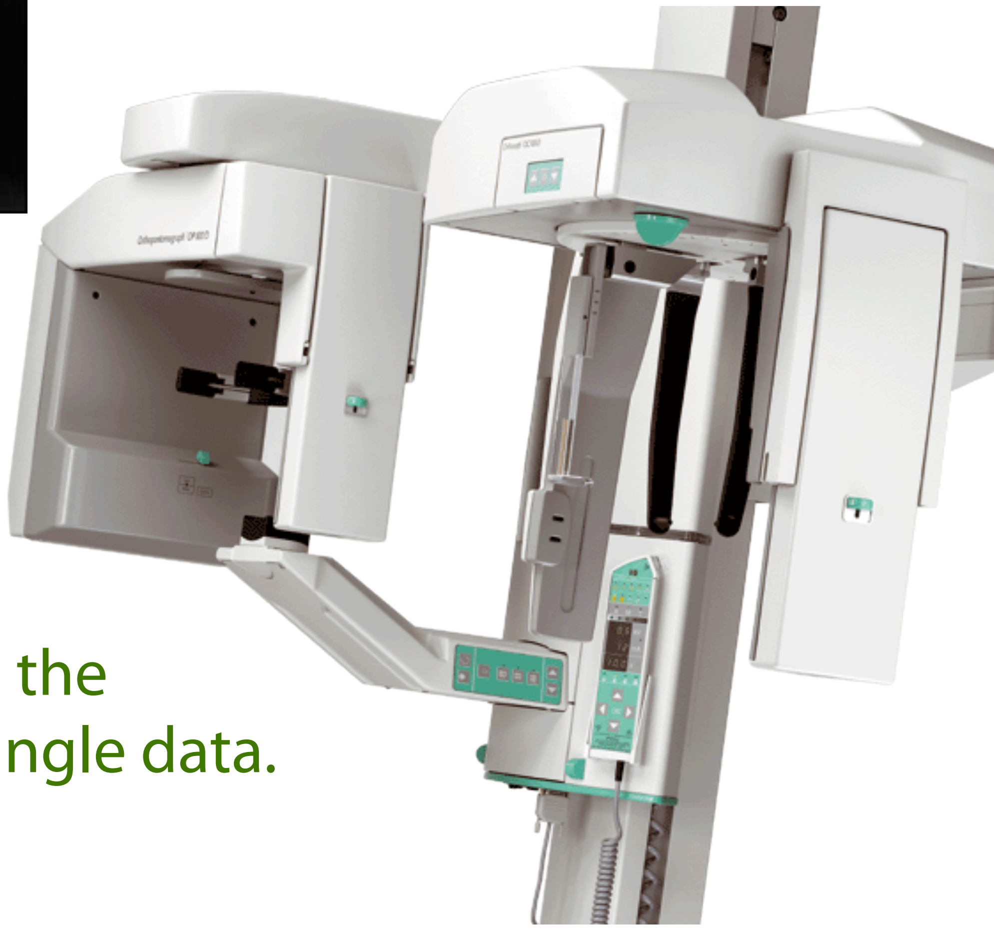
Panoramic X-ray device rotates around the head and produces a general picture

Panoramic imaging was invented by Yrjö Paatero in 1950's.



Nowadays a panoramic device is standard equipment at every dental clinic around the world.

In our project, we reprogrammed the device so that it collects limited-angle data.

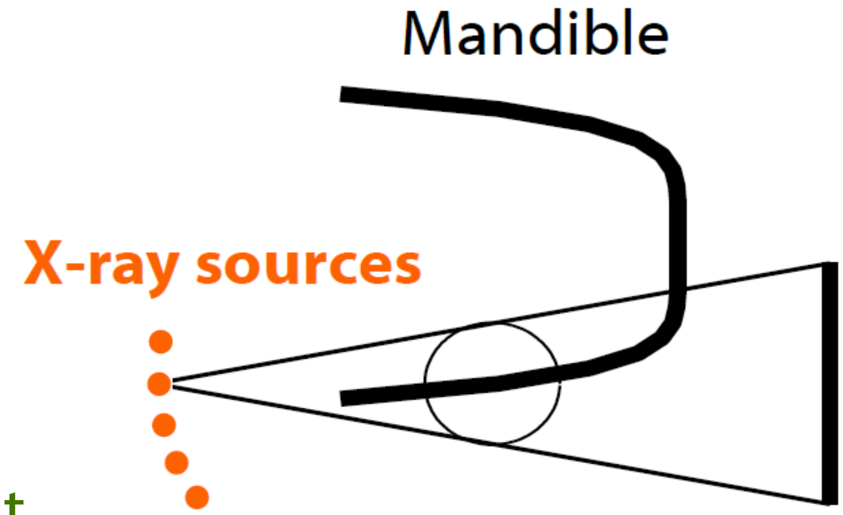


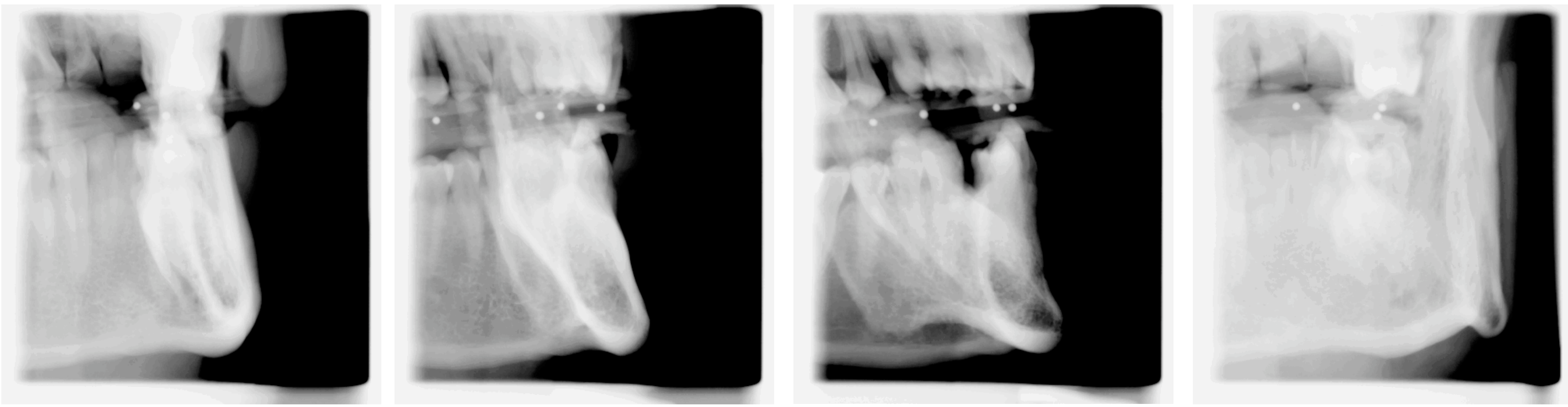
We consider the following limited angle experiment with the panoramic x-ray device:

11 projection images of the mandibular area

40 degrees aperture

1000 x 1000 pixels per image formed by a scanning movement





Limited angle reconstruction can be used for locating the mandibular nerve

This is core technology for the VT product of PaloDEx Group

Kolehmainen, Vanne, S, Järvenpää, Kaipio, Lassas and Kalke (2006)
Kolehmainen, Lassas and S (2008)
Cederlund, Kalke and Welander (2009)
Hyvönen, Kalke, Lassas, Setälä, Siltanen (submitted)

Essential history of the three projects:

Academic members: Inverse problems research groups in University of Helsinki, University of Kuopio, Helsinki University of Technology and Tampere University of Technology

Industrial members:
2001-2002 Instrumentarium Imaging and Invers Ltd
2003-2004 GE Healthcare Finland
2005-2007 PaloDEx Group

Funding by TEKES and the companies.

Outcome: 12 peer-reviewed articles, 3 patents, algorithms for 3 commercial products