Electrical impedance tomography: Backscattering and imaging of concrete

Nuutti Hyvönen

Aalto University

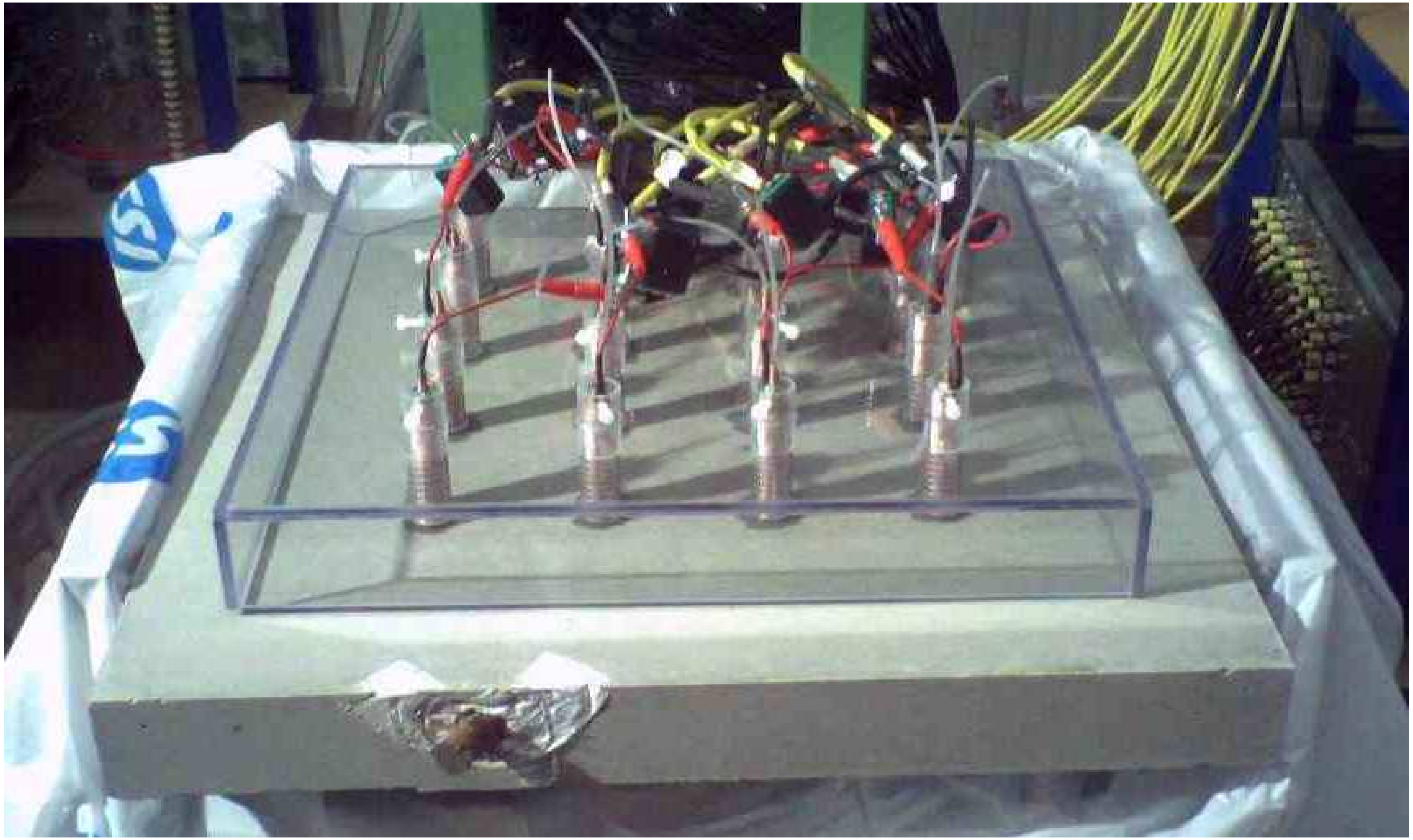
`nuutti.hyvonen@hut.fi`

joint work with Martin Hanke, Bastian Harrach, Jari Kaipio, Kimmo Karhunen, Stefanie Reusswig and Aku Seppänen.

Outline of the talk

1. Realistic electrical impedance tomography (EIT).
2. Backscattering in EIT:
   - Measurement data.
   - A uniqueness result and the convex backscattering support.
3. EIT imaging of concrete:
   - Motivation and aims.
   - Some reconstructions.

1. Realistic EIT



Complete electrode model

According to the so-called *complete electrode model*, the forward problem of EIT is as follows: Given a mean free vector of electrode currents $I \in \mathbb{C}^M$, find the electrode potentials $U \in \mathbb{C}^M$ and the electromagnetic interior potential $u \in H^1(D)$ such that

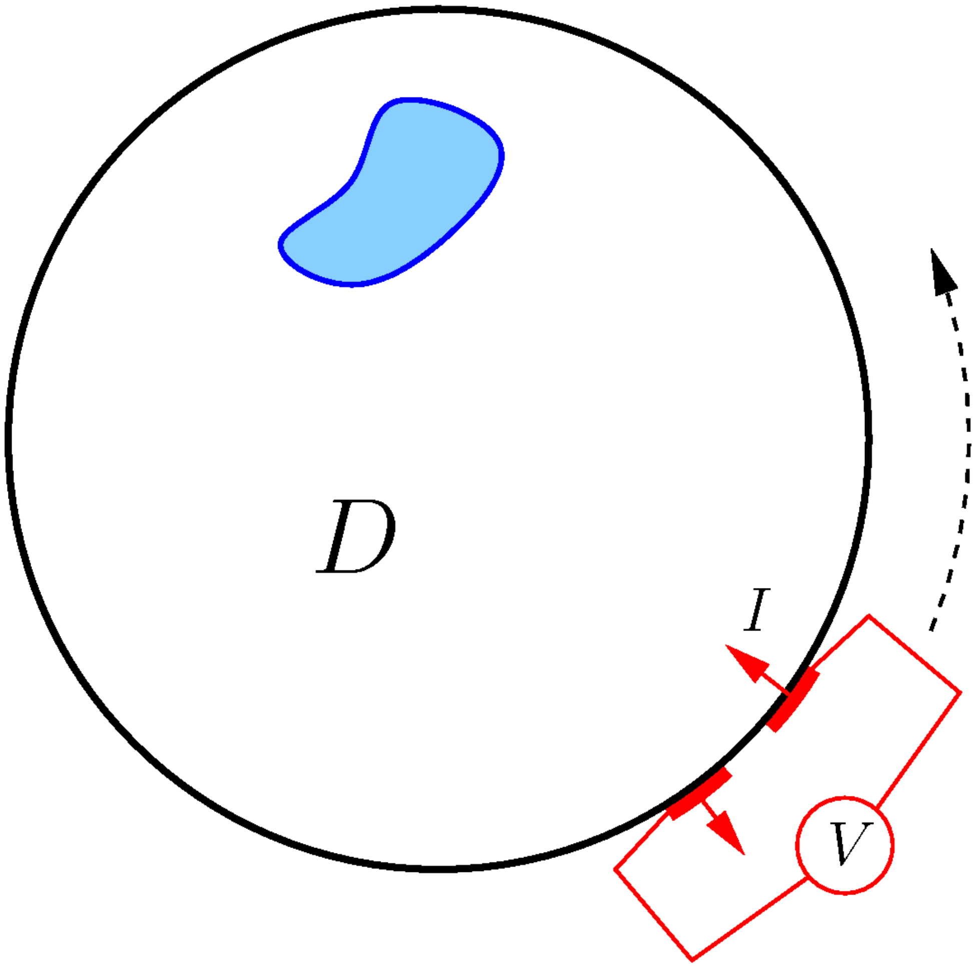
$$\begin{aligned}
&\nabla \cdot \sigma \nabla u = 0 && \text{in } D, \\
&\nu \cdot \sigma \nabla u = 0 && \text{on } \partial D \setminus (\textstyle\bigcup \overline{e}_m), \\
&u + z_m \, \nu \cdot \sigma \nabla u = U_m && \text{on } e_m, \quad m = 1, \dots, M, \\
&\int_{e_m} \nu \cdot \sigma \nabla u \, dS = I_m, && m = 1, \dots, M.
\end{aligned}$$

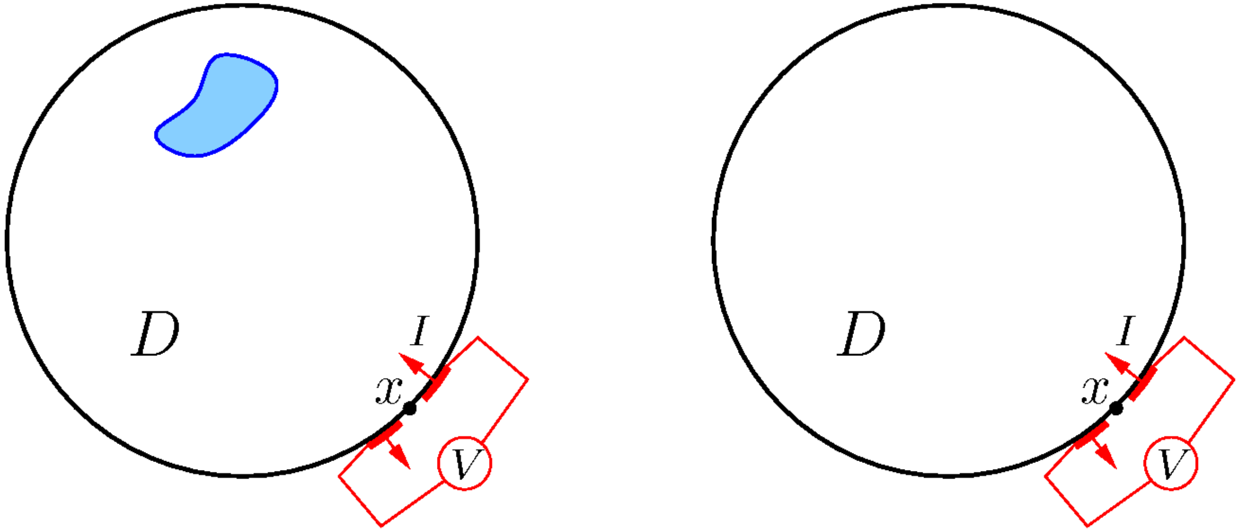
These equations define (u, U) up to the ground level of potential.

2. Backscattering in EIT

(Johannes Gutenberg–Universität Mainz)

Measurement data





Suppose that the available measurement $M_h(x)$ is the reading of the voltmeter on the left minus that on the right; here $h > 0$ is the width of the electrodes as well as the width of the gap between them. One way of defining the *backscatter data* $b : \partial D \to \mathbb{R}$ of EIT is setting

$$b(x) = \lim_{h \to 0} \frac{M_h(x)}{h^2}, \qquad x \in \partial D,$$

in the framework of the complete electrode model.

A uniqueness result

Theorem. *Let b_1 and b_2 be the backscatter data corresponding to the simply connected insulating C^2-cavities Ω_1 and Ω_2, respectively. If $b_1 = b_2$ on some open nonempty subset of ∂D, then also $\Omega_1 = \Omega_2$.*

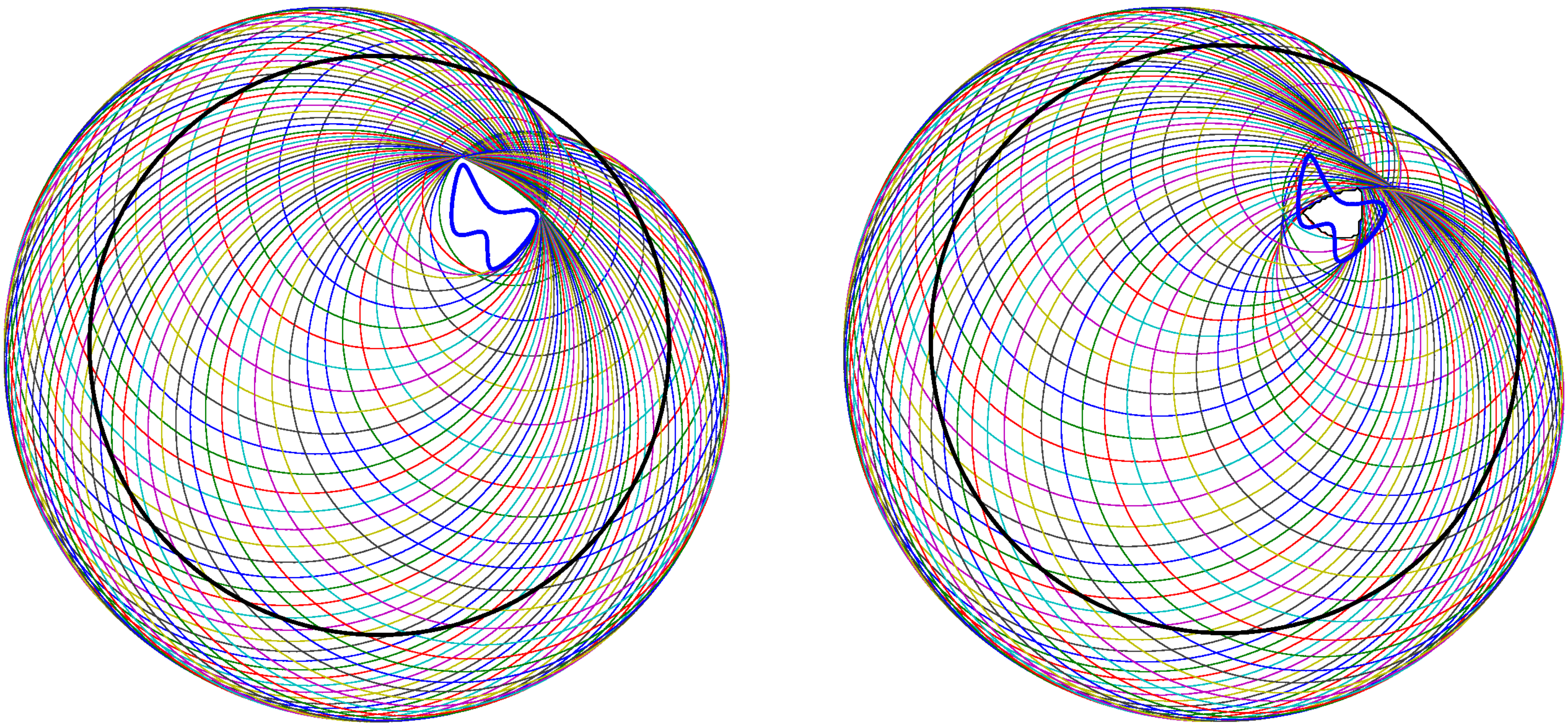
Proof. The proof is based on tools of complex analysis such as the Riemann mapping theorem for doubly connected domains and the Schwarzian derivative. □

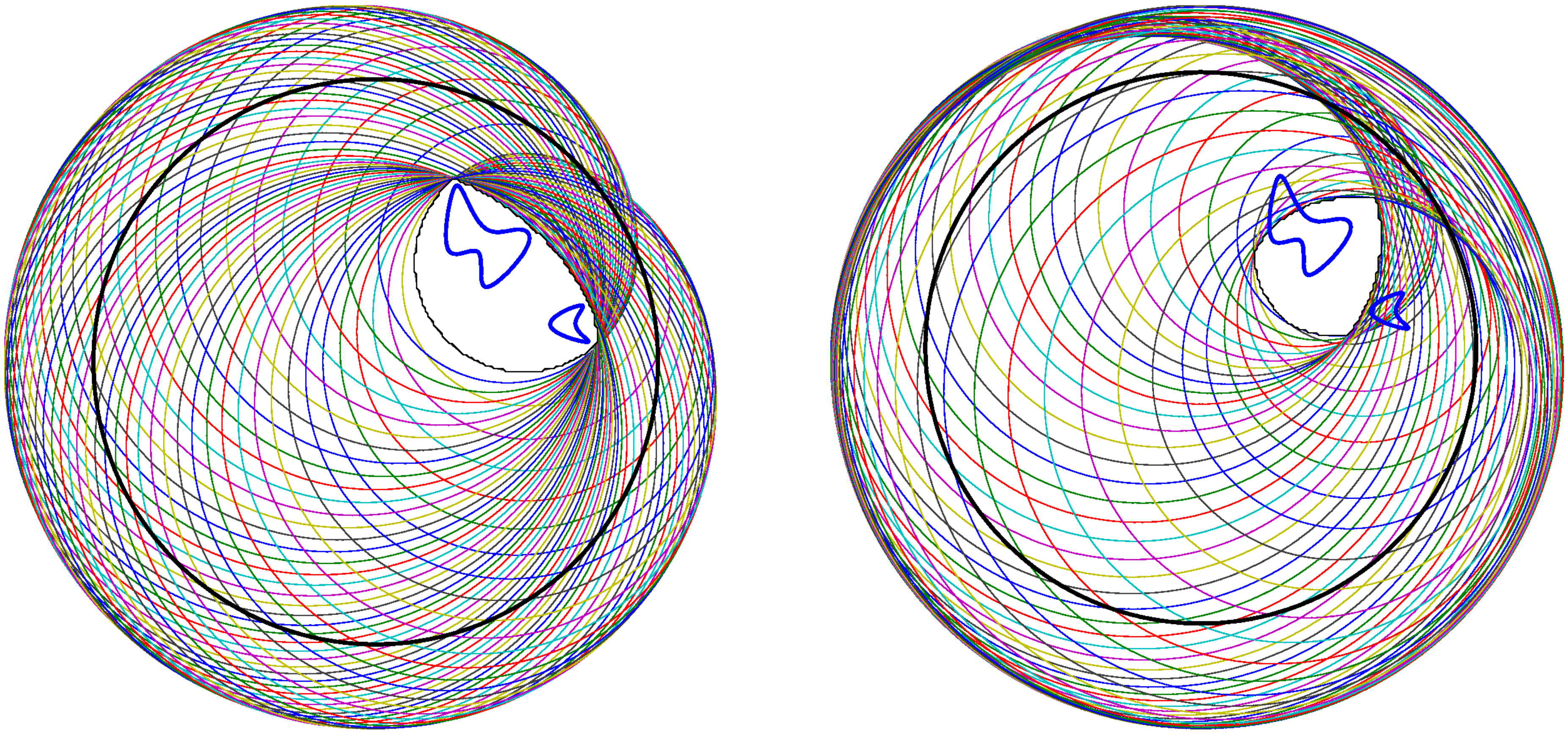
Convex backscattering support

- The backscatter data corresponding to a general compactly supported L^∞-perturbation of the conductivity is the trace of a potential that satisfies the Poisson equation in D with homogeneous Neumann data on ∂D.

- Since the corresponding source is supported in the inhomogeneity, the reconstruction problem may be recast as an inverse source problem.

- In the following, we show reconstructions of the *convex backscattering support*, i.e., of the smallest convex set that carries an electrostatic source for which the associated potential coincides with the backscatter data on the object boundary.





Related publications

M. Hanke, N. Hyvönen, and S. Reusswig, *Convex source support and its application to electric impedance tomography*, SIAM Journal on Imaging Sciences, **1**, 364–378 (2008).

M. Hanke, N. Hyvönen, and S. Reusswig, *An inverse backscatter problem for electric impedance tomography*, SIAM Journal on Mathematical Analysis, **41**, 1948-1966 (2009).

M. Hanke, N. Hyvönen, and S. Reusswig, *Convex backscattering support in electric impedance tomography*, submitted.

M. Hanke, B. Harrach, and N. Hyvönen, *Justification of point electrode models in electrical impedance tomography*, submitted.

S. Kusiak and J. Sylvester, *The scattering support*, Communications in Pure and Applied Mathematics, **56**, 1525–1548 (2003).

2. EIT imaging of concrete

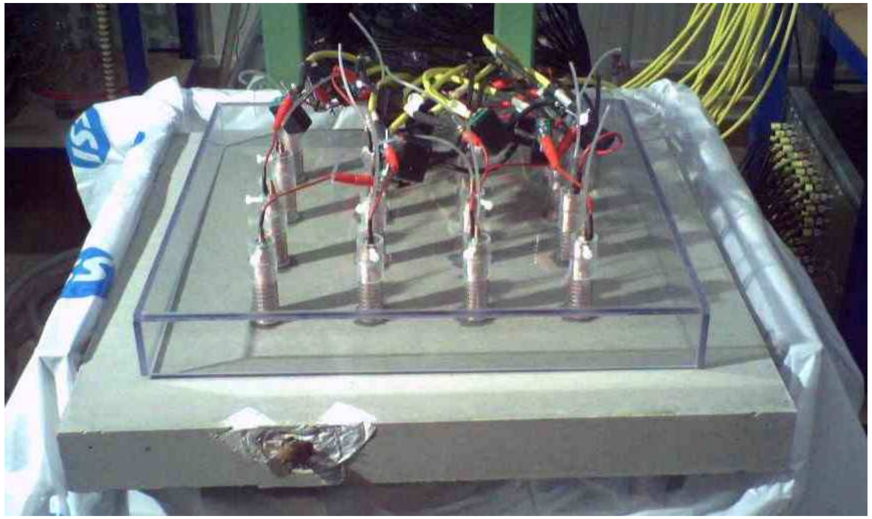
(University of Eastern Finland, Kuopio)

Motivation

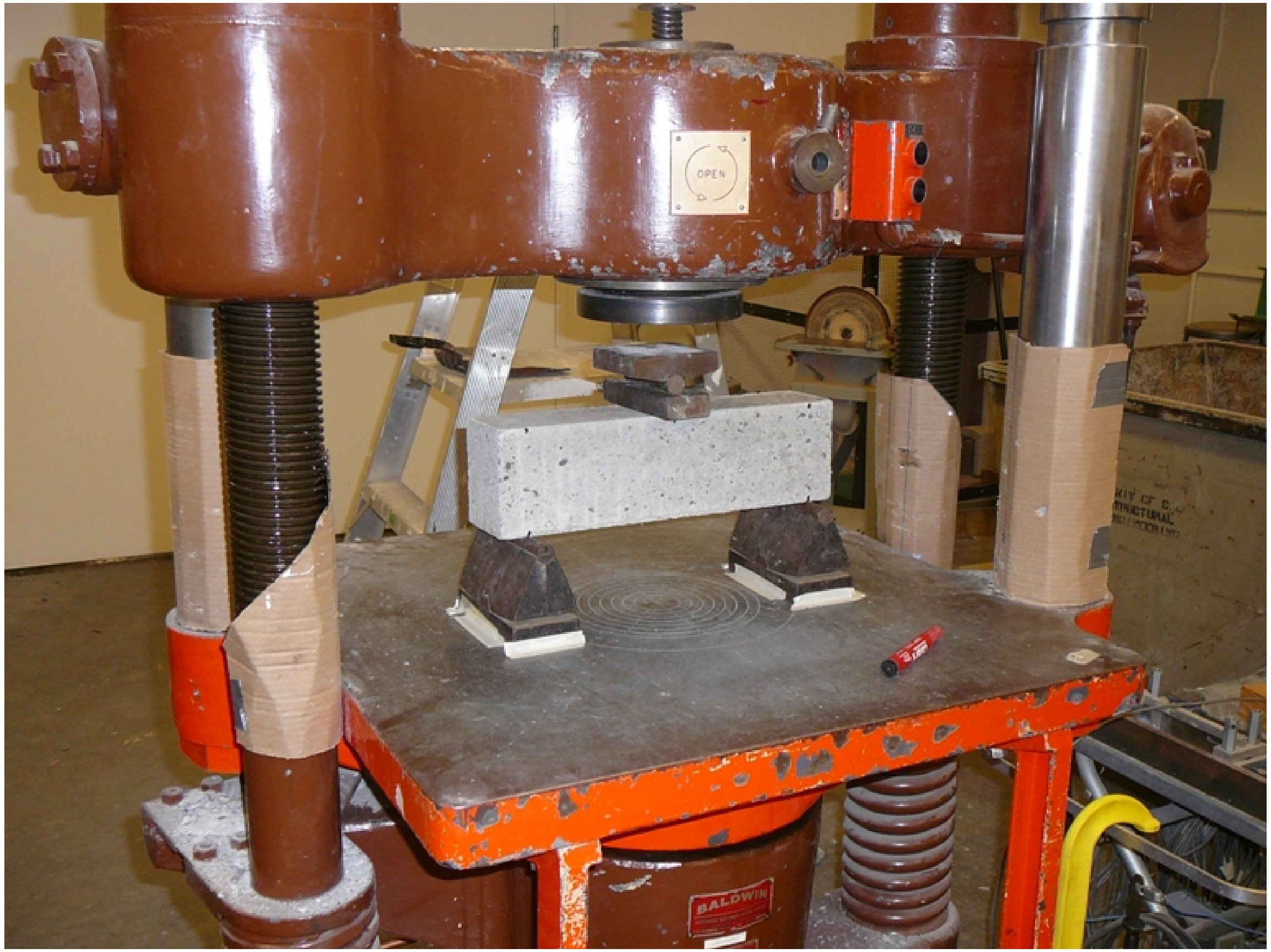
- Concrete is the most extensively used construction material in the world.
- About 7.5 cubic kilometers of concrete is cast each year.
- A $35 billion industry.
- Evaluation, repair and restoration constitute 35% of the total work volume in building industry.

Objectives

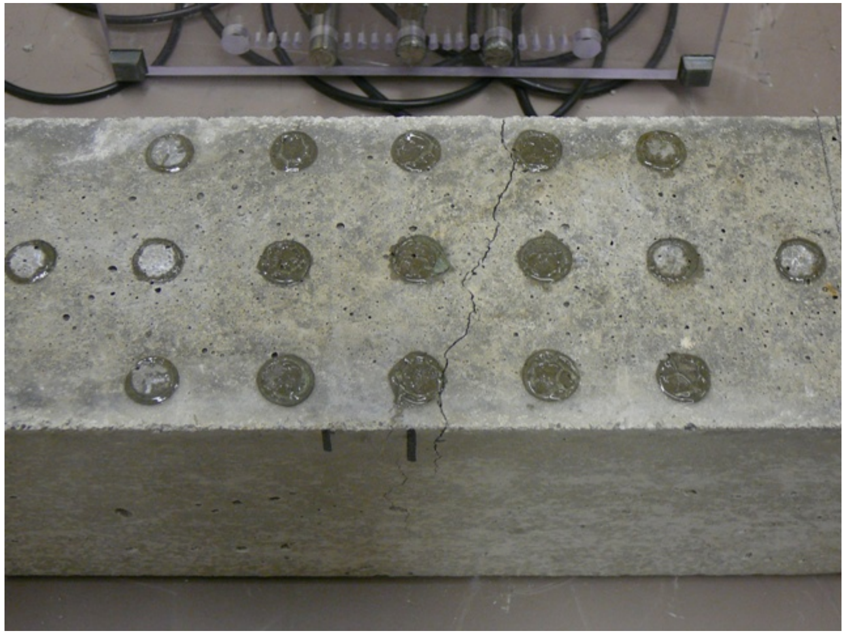
- Localization of rebars, estimation of corrosion rate.
- Detection of cracks.
- Monitoring of other properties (moisture, chlorides, carbonation, etc).

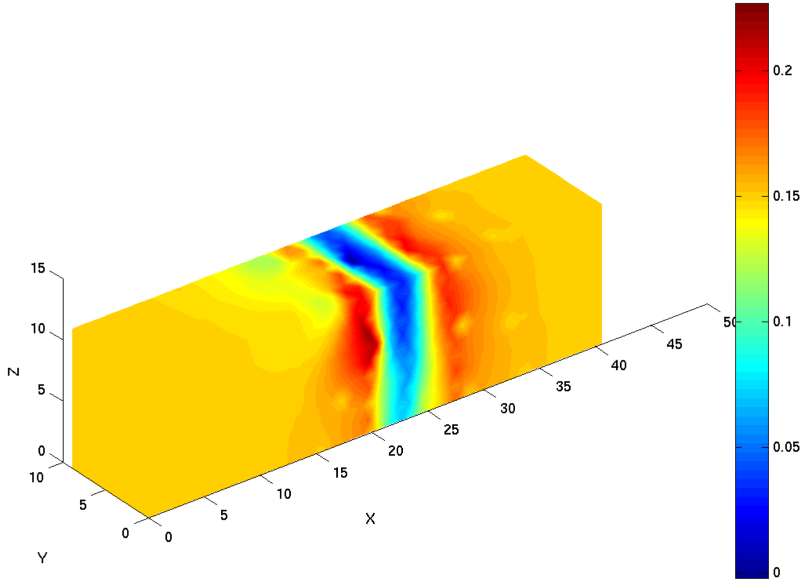


Detection of cracks

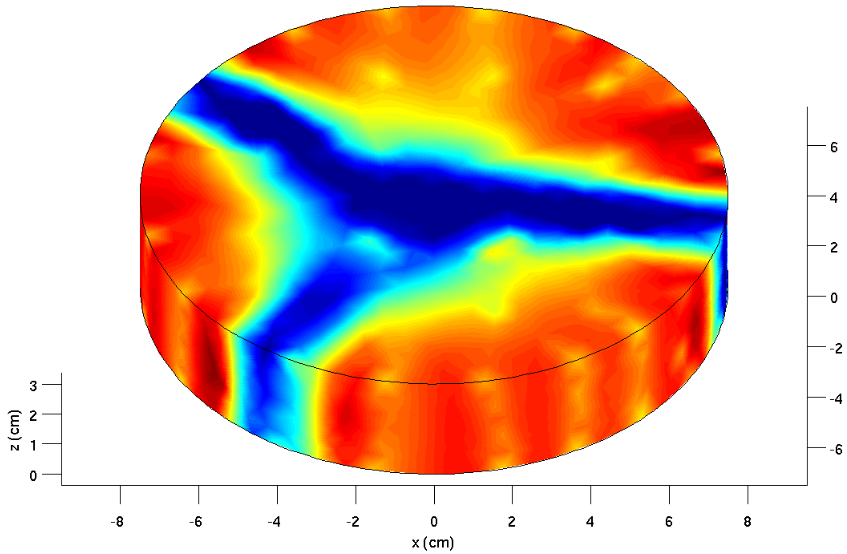


Beam of concrete

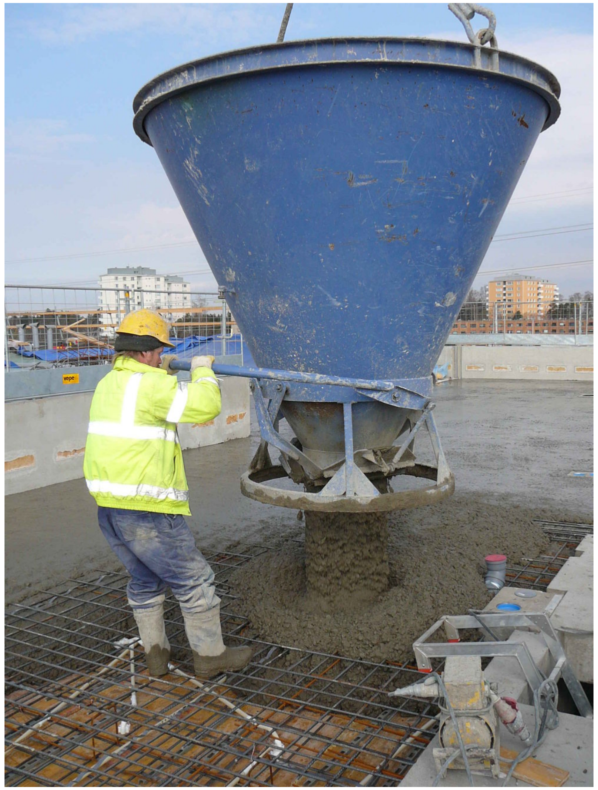
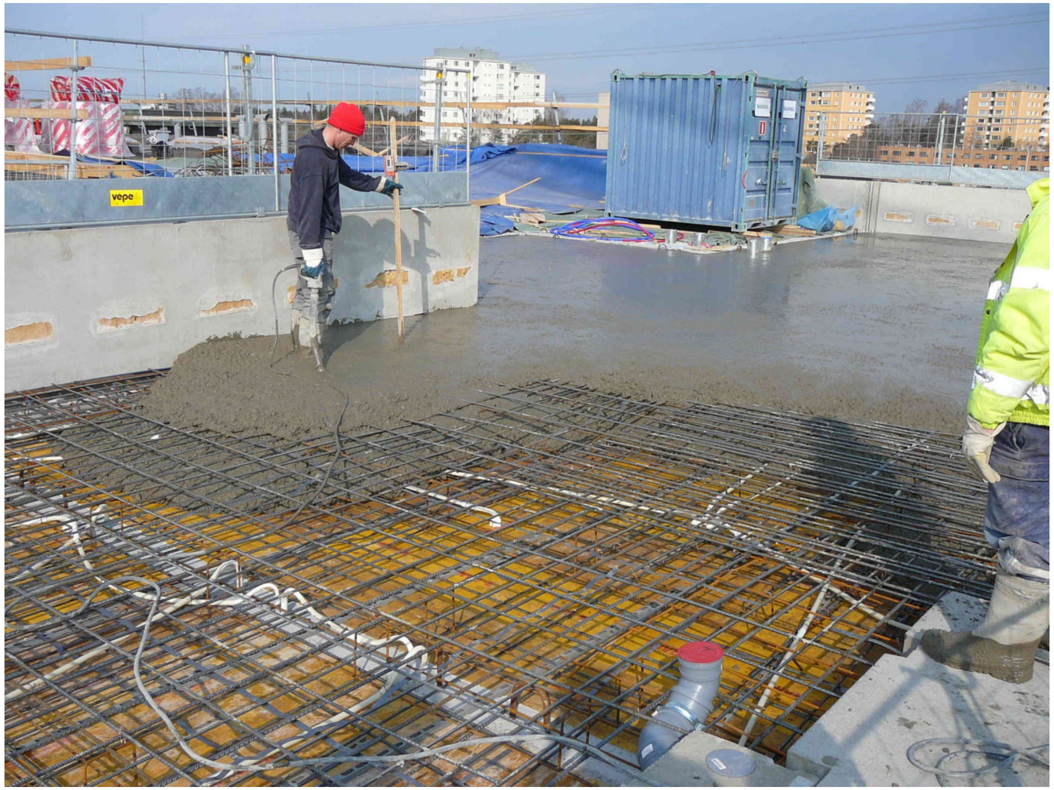




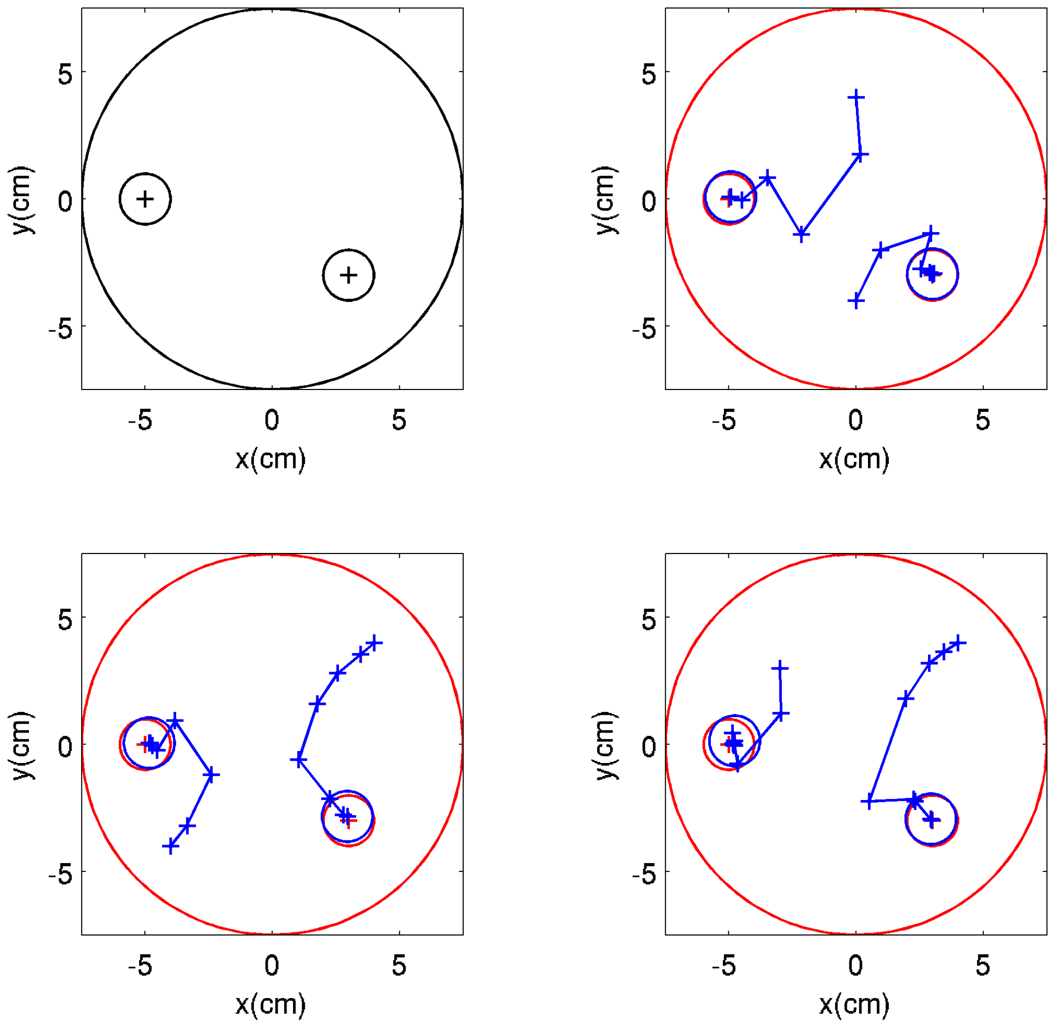
Concrete cylinder



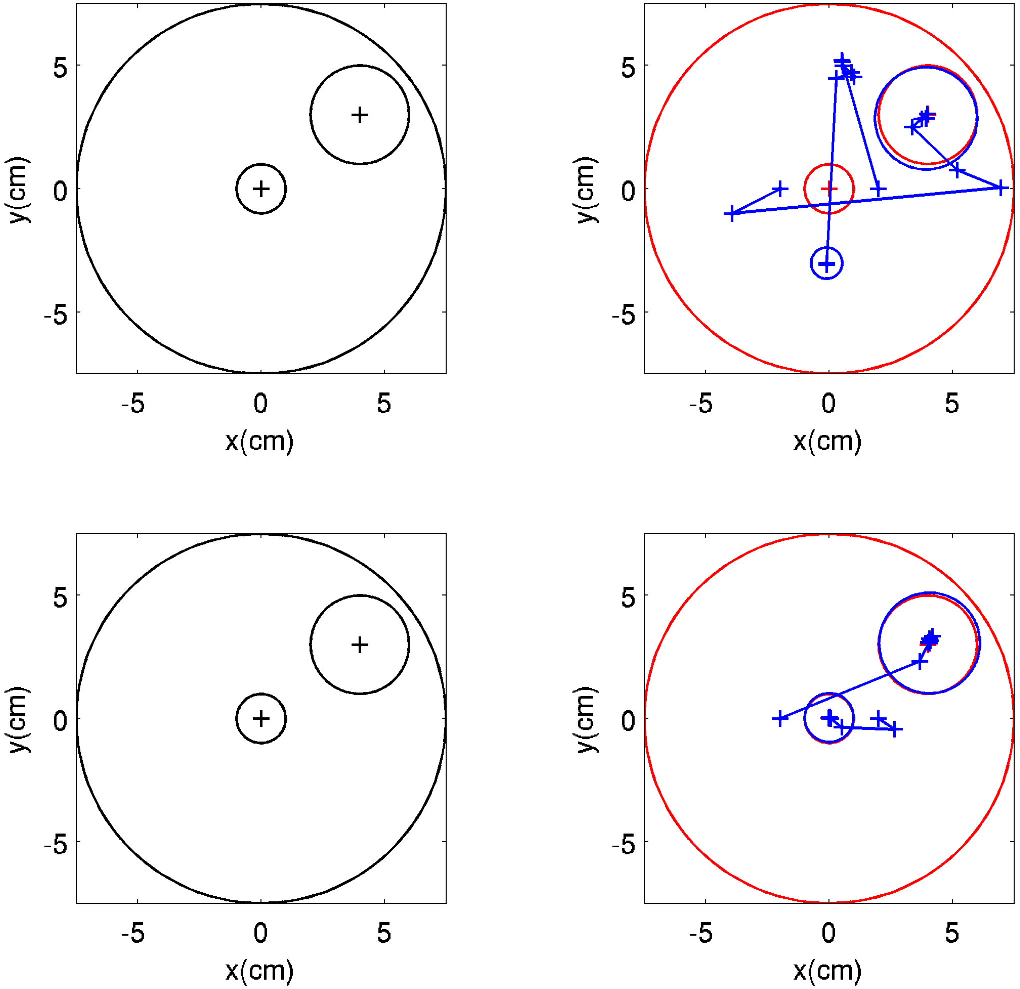
Localization of rebars

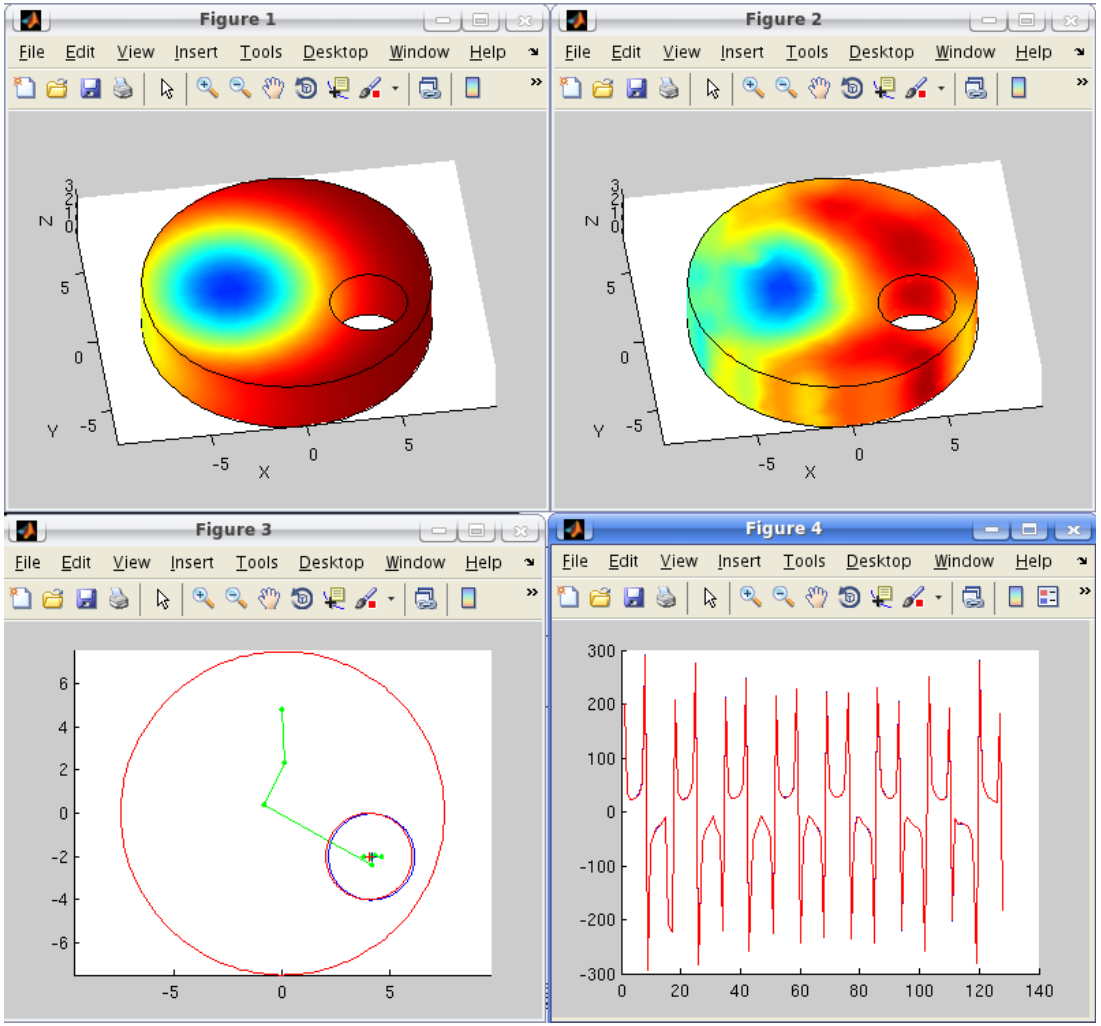
Two-and-half-dimensional simulation study



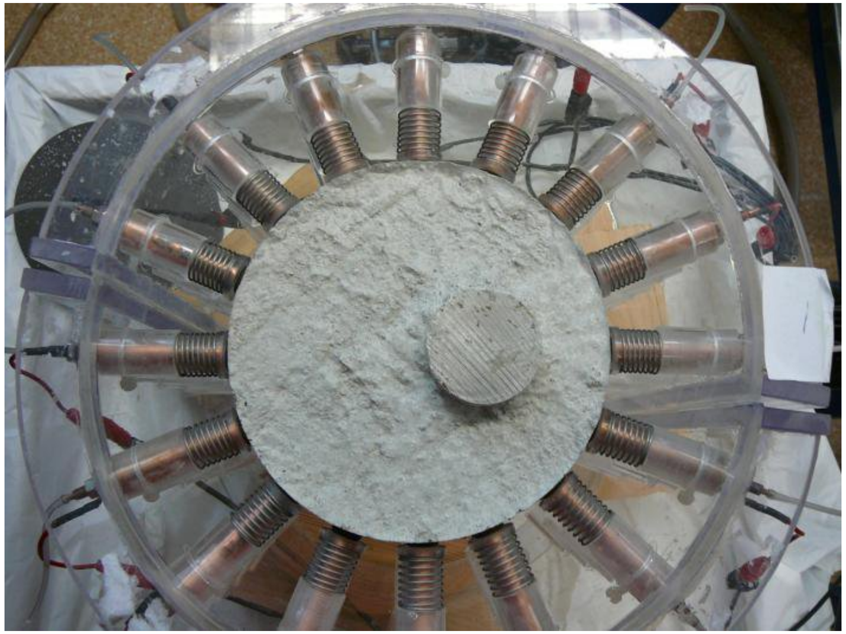
Current injection through the rebars



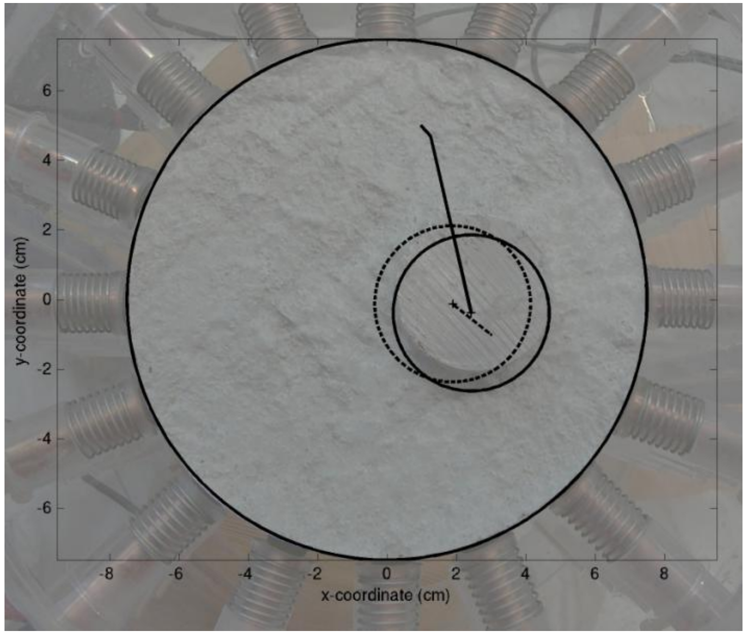
Rebar in non-homogeneous background



Real Data



Specimen



Reconstructed location

Related publications

N. Hyvönen, K. Karhunen and A. Seppänen, *Fréchet derivative with respect to the shape of an internal electrode in Electrical Impedance Tomography*, SIAM Journal on Applied Mathematics, **70**, 1878–1898 (2010).

K. Karhunen, A. Seppänen, A. Lehikoinen, P. J. M. Monteiro, and J. P. Kaipio, *Electrical resistance tomography imaging of concrete*, Cement and Concrete Research, **40**, 137–145 (2010).

K. Karhunen, A. Seppänen, A. Lehikoinen, J. Blunt, J. P. Kaipio, P. J. M. Monteiro, *Electrical resistance tomography for assessment of cracks in concrete*, submitted.

A. Kirsch, *The domain derivative and two applications in inverse scattering theory*, Inverse Problems, **9**, 81–96 (1993).


BioMedical Ultrasound
Laboratory
UNIVERSITY OF KUOPIO