

1 Introduction

The table in the following section is a re-elaboration of a table in [2] (pag. 136). Kolmogorov's axiomatic theory of probability was very shortly reviewed in the first lecture and in chapter 2 of [1]. This theory enroots the foundations of probability theory in set theory and measure theory. It is therefore useful to summarize the interpretation of basic concepts in set and probability theory.

2 The table

Notation	Interpretation in set theory	Interpretation in probability theory
ω	Element, point.	Outcome, elementary event.
Ω	Set of points.	Space of the elementary events, certain event.
\mathcal{F}	σ -algebra of the subsets.	σ -algebra of the events.
$F \in \mathcal{F}$	A set of points.	An event: $\omega \in F$ implies the occurrence of F .
$F^c = \Omega/F$	Complementary set i.e. the set of points ω not belonging to F .	The event that F does not occur.
$F_1 \cup F_2$	Union of the sets F_1 and F_2 i.e. the set of points ω belonging either to F_1 or to F_2 .	The event that <i>at least</i> F_1 or F_2 occurs.
$F_1 \cap F_2$	Intersection of the sets F_1 and F_2 i.e. the set of points ω <i>simultaneously</i> belonging to F_1 and F_2 .	The event that F_1 and F_2 simultaneously occur.
\emptyset	The empty set.	The impossible event.
$F_1 \cap F_2 = \emptyset$	F_1 and F_2 do not intersect.	The <i>simultaneous</i> occurrence of F_1 and F_2 is impossible i.e. F_1 and F_2 are <i>mutually exclusive</i> .
$F_1 + F_2$	Union of non intersecting sets F_1 and F_2 .	Occurrence of one out of two mutually exclusive events.
$F_1 \setminus F_2$	Difference of F_1 and F_2 : points belonging to F_1 but not to F_2 . It is valid to "subtract" members of a set that are not in the set as the operation has no effect.	The event that F_1 occurs but not F_2 .
$F_1 \Delta F_2 \equiv (F_1 \setminus F_2) \cup (F_2 \setminus F_1)$.	Symmetric difference of F_1 and F_2 i.e. the set of elements which are in one of the sets, but not in both.	Event that either F_1 or F_2 occurred but not. both simultaneously.
$\bigcup_{n=1}^{\infty} F_n$ $\sum_{n=1}^{\infty} F_n$	Infinite union of elements of $\{F_n\}_{n=1}^{\infty}$. Infinite union of elements of the sequence of non-intersecting sets $\{F_n\}_{n=1}^{\infty}$.	event that one of the $F_{nn=1}^{\infty}$ occurred. Event that one of the <i>mutually exclusive</i> events $\{F_n\}_{n=1}^{\infty}$ occurs.
$\bigcap_{n=1}^{\infty} F_n$	Intersection of the elements of $\{F_n\}_{n=1}^{\infty}$.	All the events $\{F_n\}_{n=1}^{\infty}$ <i>simultaneously</i> occur.
$F_n \uparrow F$.	The increasing sequence of sets F_k converges to F : i.e. $F_n \subset F_{n'}$ For any $n \leq n'$ and $F = \bigcup_{n=1}^{\infty} F_n$.	The increasingly <i>likely</i> sequence of sets F_n converges to F .
$F_n \downarrow F$	The decreasing sequence of sets F_n converges to F , i.e. $F_{n'} \subset F_n$ for any $n \leq n'$ and $F = \bigcap_{n=1}^{\infty} F_n$.	The increasingly <i>unlikely</i> sequence of sets F_n converges to F .
$\limsup F_n$.	The set $\bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} F_k$.	F_n <i>i.o.</i> the event that infinitely many of the $\{F_n\}_{n=1}^{\infty}$ occur.
$\liminf F_n$	The set $\bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} F_k$.	The event that all the events $\{F_n\}_{n=1}^{\infty}$ occur with the eventual exception of a <i>finite number</i> of them.

References

- [1] L.C. Evans, *An Introduction to Stochastic Differential Equations*, lecture notes, <http://math.berkeley.edu/~evans/>. 1
- [2] A. N. Shiryaev, *Probability*, 2nd Ed. Springer (1996), <http://books.google.com/> 1