Linear algebra and matrices II, fall 2009 Exercise 4 (30.11.-4.12.2009)
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1. Leyt A be a square matrix. Prove:

   (a) If A has a zero row, then $\det(A) = 0$.

   (b) Let B be formed by multiplying a row of A by a constant k. Then $\det(B)=k\det(A)$.

   (c) Let B be formed by adding a row of A multiplied by k to another row of A. Then $\det(B)=\det(A)$.

2. Let B be an arbitrary $n \times n$-matrix and E an elementary matrix of size $n \times n$. Prove that $\det(EB) = \det(E)\det(B)$.

3. Let A be the upper triangular matrix

$$A = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}.$$

   (a) Show that $\det(A) = adf$, the product of diagonal elements.

   (b) Show that the diagonal elements of A are the eigenvalues of A.

4. Determine the eigenvalues of

$$A = \begin{bmatrix} 2 & 4 \\ 6 & 0 \end{bmatrix}$$

   by computing the roots of the characteristic polynomial. Find a basis for the eigenspace of each eigenvalue.

5. Show that the following matrices A and B are not similar.

$$A = \begin{bmatrix} 3 & -1 \\ -5 & 7 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 1 \\ -4 & 6 \end{bmatrix}.$$

6. Let

$$A = \begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix}.$$

   (a) Determine the eigenvalues of A.

   (b) Find the eigenvectors of A.

   (c) Write A in the form $A = PDP^{-1}$ where D is diagonal.