Linear algebra and matrices II, fall 2009 Exercise 3 (23.-27.11.2009)
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1. Prove that the following functions are linear mappings and determine their matrices in standard basis.

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+y \\ x-y \end{bmatrix}, \qquad S\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x+2y \\ 3x-4y \end{bmatrix}.$$

Determine the standard matrix of the composite map $S \circ T$ as well.

2. Use counterexamples toshow that the following functions are not linear:

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x^2 \end{bmatrix}, \qquad S\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+1 \\ y-1 \end{bmatrix}.$$

3. Determine the standard matrix of the following composite linear map from $\mathbb{R}^2$ to $\mathbb{R}^2$: first rotation counterclockwise by angle $\pi/4$, then projection to y-axis and finally another rotation counterclockwise by angle $\pi/4$.

4. Show that $\vec{v}$ is an eigenvector for matrix A and determine the corresponding eigenvalue:

$$\text{(a) } A = \begin{bmatrix} -1 & 1 \\ 6 & 0 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}; \qquad \text{(b) } A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & -2 \\ 1 & 0 & 1 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}.$$

5. Show that λ is an eigenvalue of A and find some eigenvector related to λ.

$$\text{(a) } A = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}, \quad \lambda = 3; \qquad \text{(b) } A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}, \quad \lambda = -1.$$

6. Compute the following determinants.

$$\text{(a) } \begin{vmatrix} 1 & 0 & 3 \\ 5 & 1 & 1 \\ 0 & 1 & 2 \end{vmatrix}, \qquad \text{(b) } \begin{vmatrix} \cos\theta & \sin\theta & \tan\theta \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{vmatrix}.$$