Linear algebra and matrices II, fall 2009 Exercise 2 (16.-20.11.2009)
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1. Show that $w \in \text{span}(\mathcal{B})$, when

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}, \qquad w = \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix}.$$

What are the coordinates of w in the basis $\mathcal{B}$?

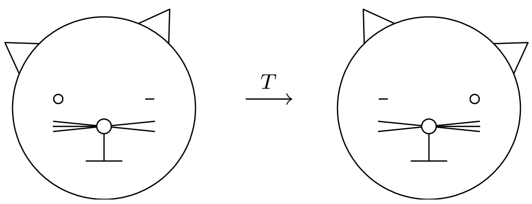
2. Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}.$$

   (a) Prove that $\mathcal{B}$ is a basis for $\mathbb{R}^2$.

   (b) What are the coordinates of the standard basis vector $\vec{e}_1 \in \mathbb{R}^2$ in the basis $\mathcal{B}$?

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map that reflects a given vector with respect to the vertical coordinate axis (see picture).



   (a) What is the matrix of T in standard basis?

   (b) What is the matrix of T in the basis $\mathcal{B}$ of problem 2 above?

4. Show that if the columns of a matrix A are linearly independent, then they form a basis for $\text{col}(A)$.

5. Let A be an invertible 2×2 matrix.

   (a) Determine $\text{null}(A)$.

   (b) Determine $\text{row}(A)$.

   (c) Determine $\text{row}(A^T)$.

6. Square matrices A and B are *similar* if there exists an invertible matrix P satisfying $P^{-1}AP = B$. In that case we denote $A \sim B$.
Show that the similarity relation satisfies

   (a) $A \sim A$,

   (b) if $A \sim B$ then $B \sim A$,

   (c) if $A \sim B$ and $B \sim C$ then $A \sim C$.

A relation satisfying (a)–(c) is called an *equivalence relaation.*