Linear algebra and matrices II, fall 2009 Exercise 1 (9.-13.11.2009)
Samuli Siltanen

1. Let V be a vector space. Prove: if $U \subset V$ and $W \subset V$ are subspaces, then the intersection $U \cap W$ is also a subspace.

2. Construct such subspaces U and W of the vector space $\mathbb{R}^2$ that the union $U \cup W$ is not a subspace of $\mathbb{R}^2$.

3. Let
$$A = \begin{bmatrix} 2 & -4 & 0 & 2 & 1 \\ -1 & 2 & 1 & 2 & 3 \\ 1 & -2 & 1 & 4 & 4 \end{bmatrix}.$$
Construct bases for the spaces row(A), col(A) and null(A).

4. Let
$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix}.$$
Construct bases for the spaces row(A), col(A) and null(A).

5. Let A be a 4×2 matrix.

   (a) Explain why the rows of A are necessarily linearly dependent.

   (b) What are the possible values of nullity(A)?

6. Do vectors
$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$
form a basis of $\mathbb{R}^4$?