Linear algebra and matrices I, fall 2009 Exercise 3 (28.9-2.10.2009)
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1. Are the following vectors linearly independent? Why?

$$\vec{v}_1 = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}.$$

2. Are the following vectors linearly independent? Why?

$$\vec{v}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 0 \\ 3 \\ 2 \\ 1 \end{bmatrix}, \quad \vec{v}_4 = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}.$$

3. Calculate the following matrix products:

$$(a) \begin{bmatrix} -2 & 1 & 3 \\ 2 & -3 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \qquad (b) \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \qquad (c) \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$$

4. In the photosynthesis reaction of plants carbon dioxide and water is transformed to glycose and oxygen. Find suitable coefficients to the chemical reaction equation

$$CO_2 + H_2O \longrightarrow C_6H_{12}O_6 + O_2.$$

5. Compute the matrix product AB in two different ways: directly from definition and by utilizing the block strucure.

$$A = \left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{array}\right], \qquad B = \left[\begin{array}{c|cc} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & 5 & 4 \\ -2 & 3 & 2 \end{array}\right].$$

Do you get the same result? (It should be the same.)

6. Show that a system of linear equations with augmented matrix $[A \,|\, \vec{b}]$ is consistent if and only if vector $\vec{b}$ is a linear combination of the columns of A.