

1. Are the following vectors linearly independent? Why?

$$\vec{v}_1 = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}.$$

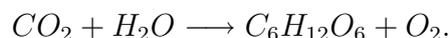
2. Are the following vectors linearly independent? Why?

$$\vec{v}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 0 \\ 3 \\ 2 \\ 1 \end{bmatrix}, \quad \vec{v}_4 = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}.$$

3. Calculate the following matrix products:

$$(a) \begin{bmatrix} -2 & 1 & 3 \\ 2 & -3 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \quad (b) \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad (c) \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$$

4. In the photosynthesis reaction of plants carbon dioxide and water is transformed to glucose and oxygen. Find suitable coefficients to the chemical reaction equation



5. Compute the matrix product  $AB$  in two different ways: directly from definition and by utilizing the block structure.

$$A = \left[ \begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{array} \right], \quad B = \left[ \begin{array}{c|ccc} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & 5 & 4 \\ -2 & 3 & 2 \end{array} \right].$$

Do you get the same result? (It should be the same.)

6. Show that a system of linear equations with augmented matrix  $[A | \vec{b}]$  is consistent if and only if vector  $\vec{b}$  is a linear combination of the columns of  $A$ .