Linear algebra and matrices I, syksy 2009 Exercise 1 (14.-18.9.2009)
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1. Let $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^n$ and $c, d \in \mathbb{R}$. Prove that

$$\begin{aligned}
&c(\vec{u}+\vec{v}) = c\vec{u} + c\vec{v},\\
&c(d\vec{u}) = (cd)\vec{u},\\
&\vec{u}\cdot(\vec{v}+\vec{w}) = \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w},\\
&\vec{u}\cdot\vec{u} \geq 0,\\
&\vec{u}\cdot\vec{u} = 0 \text{ if and only if } \vec{u} = 0.
\end{aligned}$$

2. Consider the lines in $\mathbb{R}^2$ determined by the equations

$$\text{(a) } x_2 = 3x_1 - 1, \qquad \text{(b) } 3x_1 + 2x_2 = 5.$$

Write both lines in vector form $\vec{x} = \vec{p} + t\vec{d}$.

3. Consider the plane in $\mathbb{R}^3$ that contains the points

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}.$$

Write the plane in vector form $\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$.

4. Which of the following equations are linear? Why?

(a) $\sqrt{2}x + \pi^2 y - (\log \pi^3)z = 1$, (b) $4y + e^z = 6$, (c) $x_1 + 2x_2 = 4 + x_4 - x_5$.

5. Show that each of the elementary row operations can be inverted.

6. In the picture below, thin lines depict pixels and thick lines depict X-rays. The length of the side of the pixel is one. There is an unknown X-ray attenuation coefficient, denoted by x_j, $j = 1, 2, \ldots, 9$, corresponding to each pixel. Each X-ray produces a measurement $m_k = \ell_{k1}x_1 + \ell_{k2}x_2 + \ldots + \ell_{k9}x_9$, where $1 \leq k \leq 6$ ja ℓ_{kj} ios the length that ray number k travels inside pixel number j. Write the measurement as a set of linear equations concerning the variables $x_1, x_2, \ldots, x_9$. (You can number the pixels and X-rays in any order you like.)