The infinite Hilbert matrix on spaces of analytic functions

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The (finite) Hilbert matrix is arguably one of the single most well-known matrices in mathematics. The infinite Hilbert matrix $\mathcal{H}$ was introduced by David Hilbert around 120 years ago in connection to his double series theorem. It can be interpreted as a linear operator on spaces of analytic functions by its action on their Taylor coefficients. The boundedness of $\mathcal{H}$ on the Hardy spaces H^p for $1 < p < \infty$ and Bergman spaces A^p for $2 < p < \infty$ was established by Diamantopoulos and Siskakis. The exact value of the operator norm of $\mathcal{H}$ acting on the Bergman spaces A^p for $4 \leq p < \infty$ was shown to be $\frac{\pi}{\sin(2\pi/p)}$ by Dostanic, Jevtic and Vukotic in 2008. The case $2 < p < 4$ was an open problem until in 2018 it was shown by Bozin and Karapetrovic that the norm has the same value also on the scale $2 < p < 4$. In this talk, we review some of the old results and consider the still partly open problem regarding the value of the norm on weighted Bergman spaces. We also consider a generalised Hilbert matrix operator and its (essential) norm. The talk is partly based on a joint work with Mikael Lindström, David Norrbo and Niklas Wikman (Åbo Akademi).