Spectral gaps for random covers of hyperbolic surfaces

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Based on joint work with Michael Magee.

We study the spectrum of the Laplacian for finite-area hyperbolic surfaces, focusing on the *spectral gap*, i.e. the smallest non-zero element of the spectrum. The spectral gap can be viewed as a measure of how highly connected a surface is, providing control over its diameter and Cheeger constant. It controls the rate of mixing of the geodesic flow and error terms in geodesic counting. For large genus compact surfaces, $\frac{1}{4}$ is the asymptotically optimal spectral gap.

We show that for any $\epsilon > 0$, a uniformly random Riemannian cover of a fixed non-compact hyperbolic surface has no new Laplacian eigenvalues below $\frac{1}{4} - \epsilon$ with probability tending to 1 as $n \to \infty$. As a consequence we prove the existence of a sequence of compact hyperbolic surfaces $\{X_i\}$ with genera $g_i \to \infty$ as $i \to \infty$ and $\lambda_1(X_i) \to \frac{1}{4}$, confirming a conjecture of Buser.