On a class of elliptic problems with critical growth in the gradient

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Abstract

This talk focus on the existence and multiplicity of solutions for a boundary value problem of the form

$$\begin{cases} -\Delta u = c(x)u + \mu(x)|\nabla u|^2 + h(x), & \text{in } \Omega, \\ \quad u = 0, & \text{on } \partial\Omega. \end{cases}$$

Solutions are searched in the function space $H_0^1(\Omega) \cap L^\infty(\Omega)$ where $\Omega \subset \mathbb{R}^N$, $N \geq 2$, is a bounded domain with smooth boundary. It is assumed that $c,\ h$ belong to $L^p(\Omega)$ for some $p > N/2$ and μ belongs to $L^\infty(\Omega)$.

In the case where $c(x) \leq \alpha_0 < 0$, now referred to as the *coercive case*, this problem has been studied since the 80's and the existence of a unique solution is the rule. Recently, other cases (in particular assuming that $c(x) \geq 0$ or that $c(x)$ changes sign) started to be considered. We shall present some of the main contributions in these *non-coercive cases*. We will see that both existence and uniqueness may now be lost.

The talk is based in joints works with Colette De Coster (Université Polytechnique des Hauts-de-France, Valenciennes (France)) and Louis Jeanjean (Université de Franche-Comté, Besançon (France)).