Galactic dynamics – Problem set 4. Spring 2023

*The answers should be returned by **Thursday (30.3) 4pm (16.00) in Moodle**, link through the official course homepage. A problem set help session will be held on **Thursday (23.3) at 14.15-16.00 in Room D115, Physicum**. The correct solutions will appear in Moodle after the due date.*

1. Prove that the density of a spherical, ergodic, self-consistent stellar system must decrease outward. Hint: in the integral for ρ make Φ the integration variable.

2. Show that in a frame that rotates with constant angular velocity $\mathbf{\Omega}$ with the effective potential $\Phi_{\text{eff}} = \Phi - \frac{1}{2}|\mathbf{\Omega} \times \mathbf{r}|^2$, the collisionless Boltzmann equation can be written as:
$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - [2(\mathbf{\Omega} \times \mathbf{v}) + \nabla \Phi_{\text{eff}}] \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

3. Show that a Hernquist model with constant anisotropy $\beta = \frac{1}{2}$ has
$$N(E) = \frac{3a}{GM}\tilde{\varepsilon}^2 \left(\frac{1}{\tilde{\varepsilon}} - 1\right)^2$$
where $\tilde{\varepsilon} = \epsilon a/(GM)$ and M and a are the mass and the scale radius of the Hernquist sphere.

4. A self-consistent stellar system has an ergodic DF and a power-law density profile $\rho = \rho_0 (r_0/r)^\alpha$ with $1 < \alpha < 3$. Show that the velocity dispersion is given by
$$\overline{v_r^2}(r) = \frac{2\pi G \rho_0 r_0^\alpha r^{2-\alpha}}{(3-\alpha)(\alpha-1)} \quad (\alpha \neq 2).$$
What does this formula become in the case $\alpha = 2$ of the singular isothermal sphere?

5. Prove that the following DF generates a stellar system in which the density distribution is that of a homogeneous sphere of density ρ and radius a:
$$f(E, L) = \frac{9}{16\pi^4 G\rho a^5} \frac{1}{\sqrt{L^2/a^2 + \frac{4}{3}\pi G\rho a^2 - 2E}} \quad \left(L^2 < \frac{4}{3}\pi G \rho a^4\right).$$
Here it is understood that $f = 0$ when the argument in the square root is not positive, the DF is normalised so that $\int d^3\vec{x} d^3\vec{v} f = 1$, the potential $\Phi = 0$ at $r = 0$, and the system is isolated.