Galactic dynamics – Problem set 3. Spring 2023

The answers should be returned by ***Thursday (2.3) 4pm (16.00) in Moodle****, link through the official course homepage. A problem set help session will be held on* ***Thursday (23.2) at 14.15-16.00 in Room D115, Physicum****. The correct solutions will appear in Moodle after the due date.*

1. Show that in a spherical potential the vertical and circular frequencies ν and Ω (equation 3.79 in the lecture notes) are equal.

2. Prove, that at any point in an axisymmetric system at which the local density is negligible, the epicycle, vertical and circular frequencies κ, ν, and Ω (equation 3.79 in the lecture notes) are related by $\kappa^2 + \nu^2 = 2\Omega^2$.

3. Show that in spherical polar coordinates the Lagrangian for motion in the potential $\Phi(\mathbf{x})$ is
$$\mathcal{L} = \frac{1}{2}\left[\dot{r}^2 + (r\dot{\theta})^2 + (r\sin\theta\dot{\phi})^2\right] - \Phi(\mathbf{x})$$
Hence show that the momenta p_θ and p_ϕ are related to the magnitude and z-component of the angular-momentum vector $\mathbf{L}$ by
$$p_\phi = L_z \quad ; \quad p_\theta^2 = L^2 - \frac{L_z^2}{\sin^2\theta}$$

4. Prove that the fictitious time τ in the Burdet-Heggie regularisation is related to the eccentric anomaly η by $\tau = (T_r/2\pi a)\eta + constant$, if the motion is bound $E_2 < 0$ and the external field is $\mathbf{g} = 0$.

5. Show that the leapfrog integrator (equation 3.166a) is second-order accurate, in the sense that the errors in $\mathbf{q}$ and $\mathbf{p}$ after a timestep h are of the order $O(h^3)$.