Computational light scattering (PAP315)

Lecture 1

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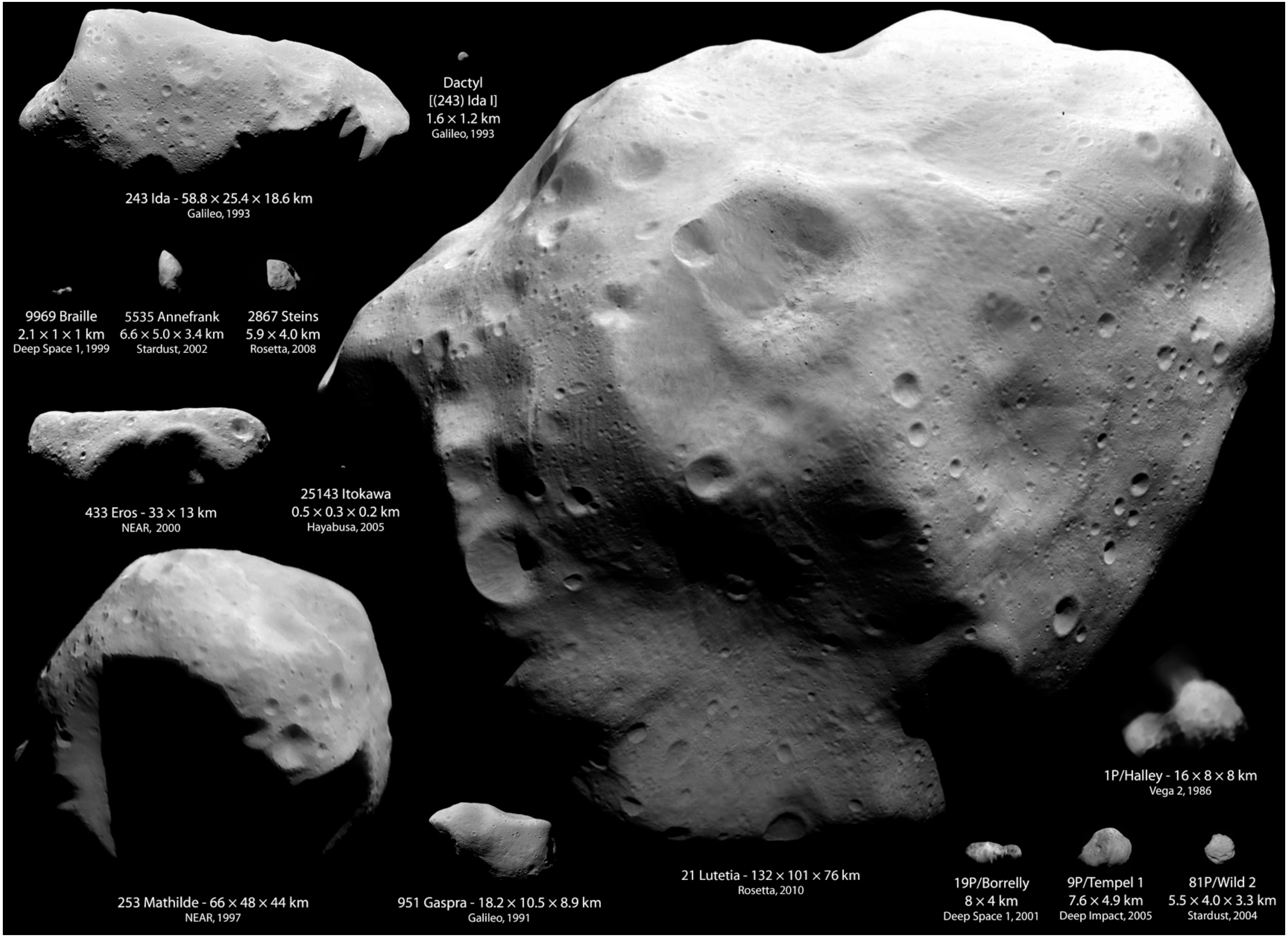
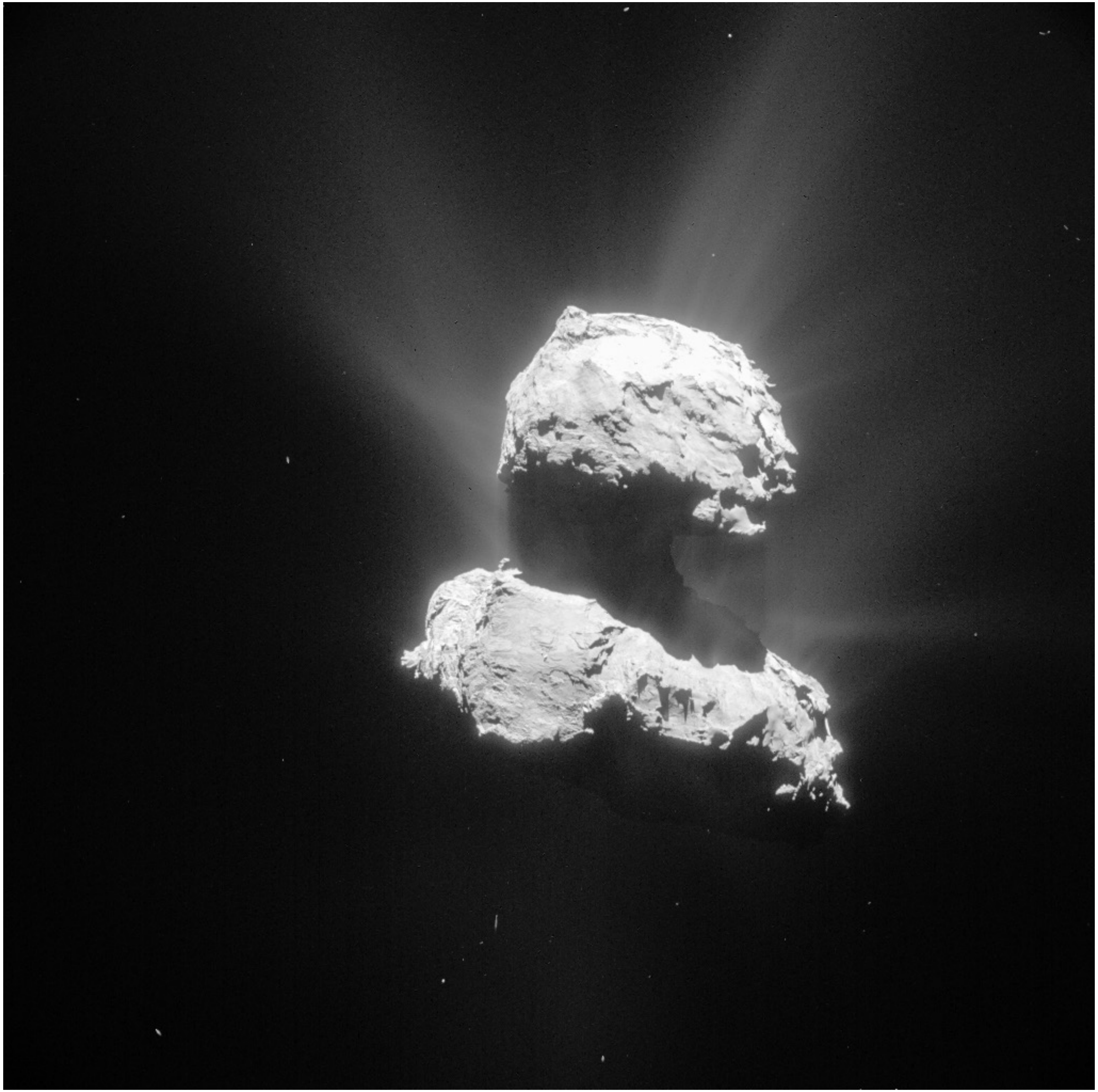
Introduction

- Physical characterization of astronomical objects (e.g., surfaces of airless planetary objects)
- Direct problem of light scattering by particles with varying particle size, shape, refractive index, and spatial distribution
- Inverse problem based on astronomical observations and/or experimental measurements
- Plane of scattering, scattering angle, solar phase angle, degree of linear polarization



Dactyl
[(243) Ida I]
1.6 × 1.2 km
Galileo, 1993
243 Ida - 58.8 × 25.4 × 18.6 km
Galileo, 1993
9969 Braille
2.1 × 1 × 1 km
Deep Space 1, 1999
5535 Annefrank
6.6 × 5.0 × 3.4 km
Stardust, 2002
2867 Steins
5.9 × 4.0 km
Rosetta, 2008
433 Eros - 33 × 13 km
NEAR, 2000
25143 Itokawa
0.5 × 0.3 × 0.2 km
Hayabusa, 2005
253 Mathilde - 66 × 48 × 44 km
NEAR, 1997
951 Gaspra - 18.2 × 10.5 × 8.9 km
Galileo, 1991
21 Lutetia - 132 × 101 × 76 km
Rosetta, 2010
1P/Halley - 16 × 8 × 8 km
Vega 2, 1986
19P/Borrelly
8 × 4 km
Deep Space 1, 2001
9P/Tempel 1
7.6 × 4.9 km
Deep Impact, 2005
81P/Wild 2
5.5 × 4.0 × 3.3 km
Stardust, 2004



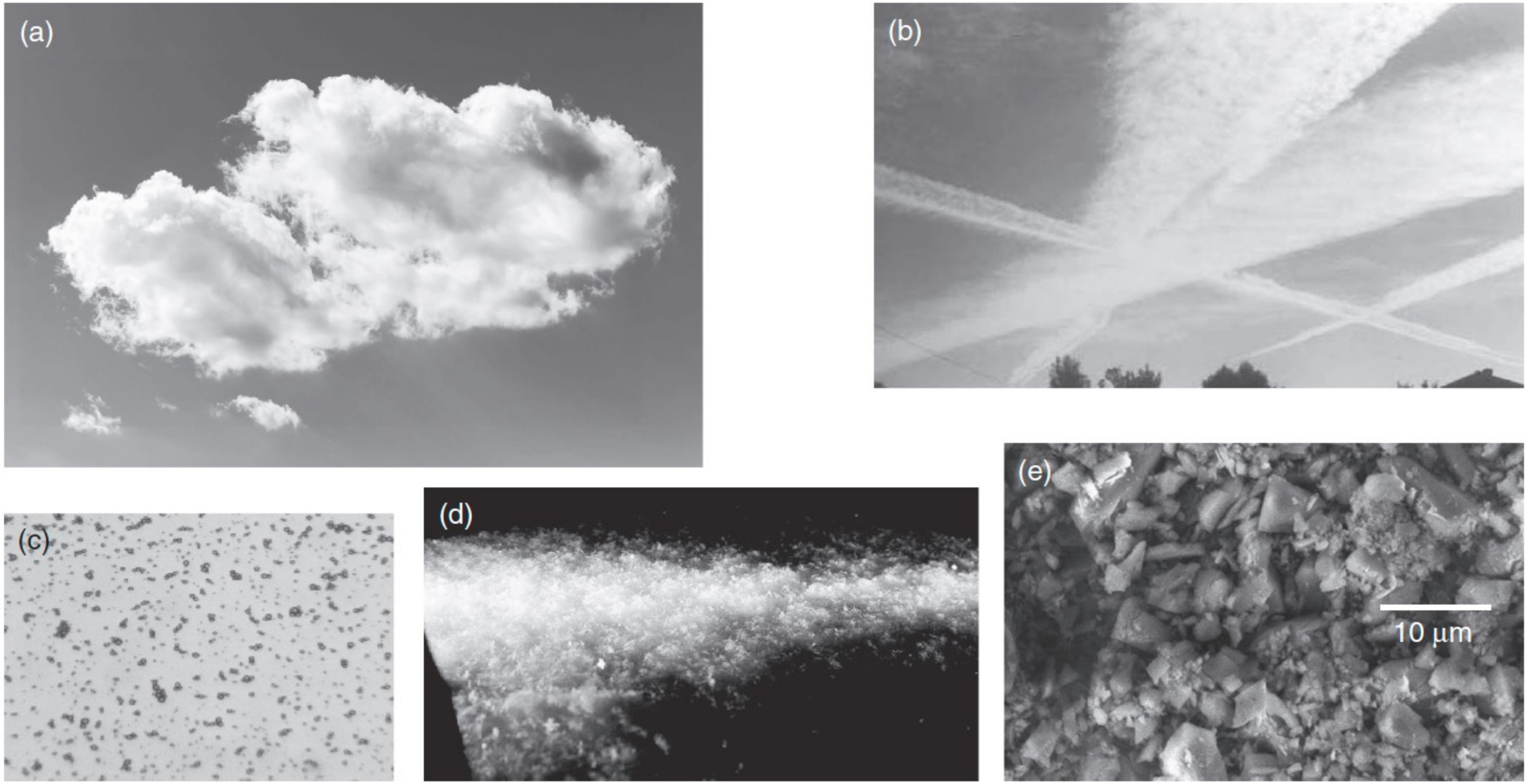


FIGURE 3.12 Examples of discrete random media. (a) A terrestrial liquid-water cloud. (b) Jet-engine condensation trail clouds. (c) A particle suspension. (d) A fresh snow surface. (e) A powder surface composed of feldspar particles.

(d) After Peltoniemi *et al.* (2009).

(e) After Shkuratov *et al.* (2006).

Mishchenko, PSPS 2015

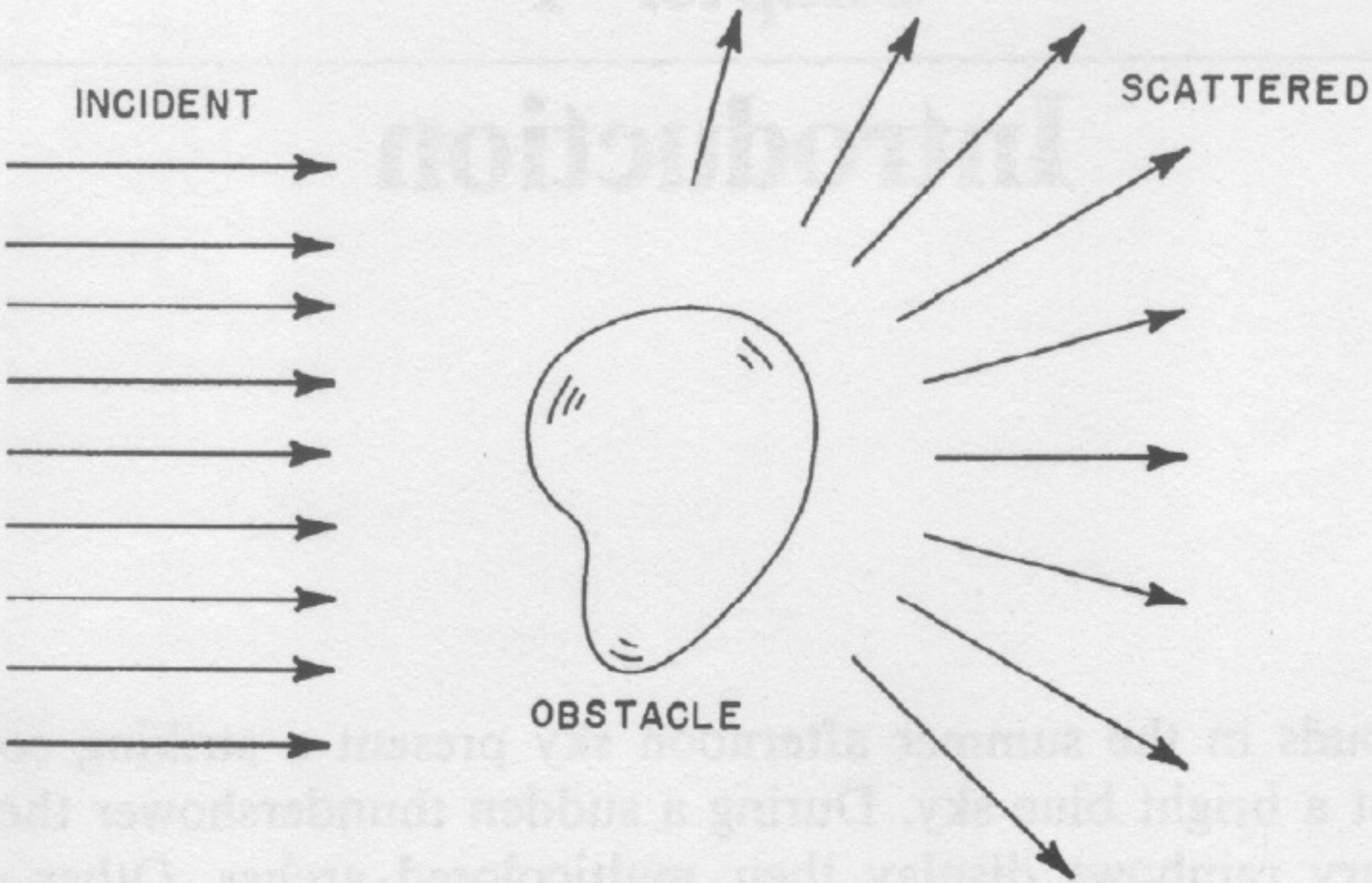


Figure 1.1 Scattering by an obstacle.

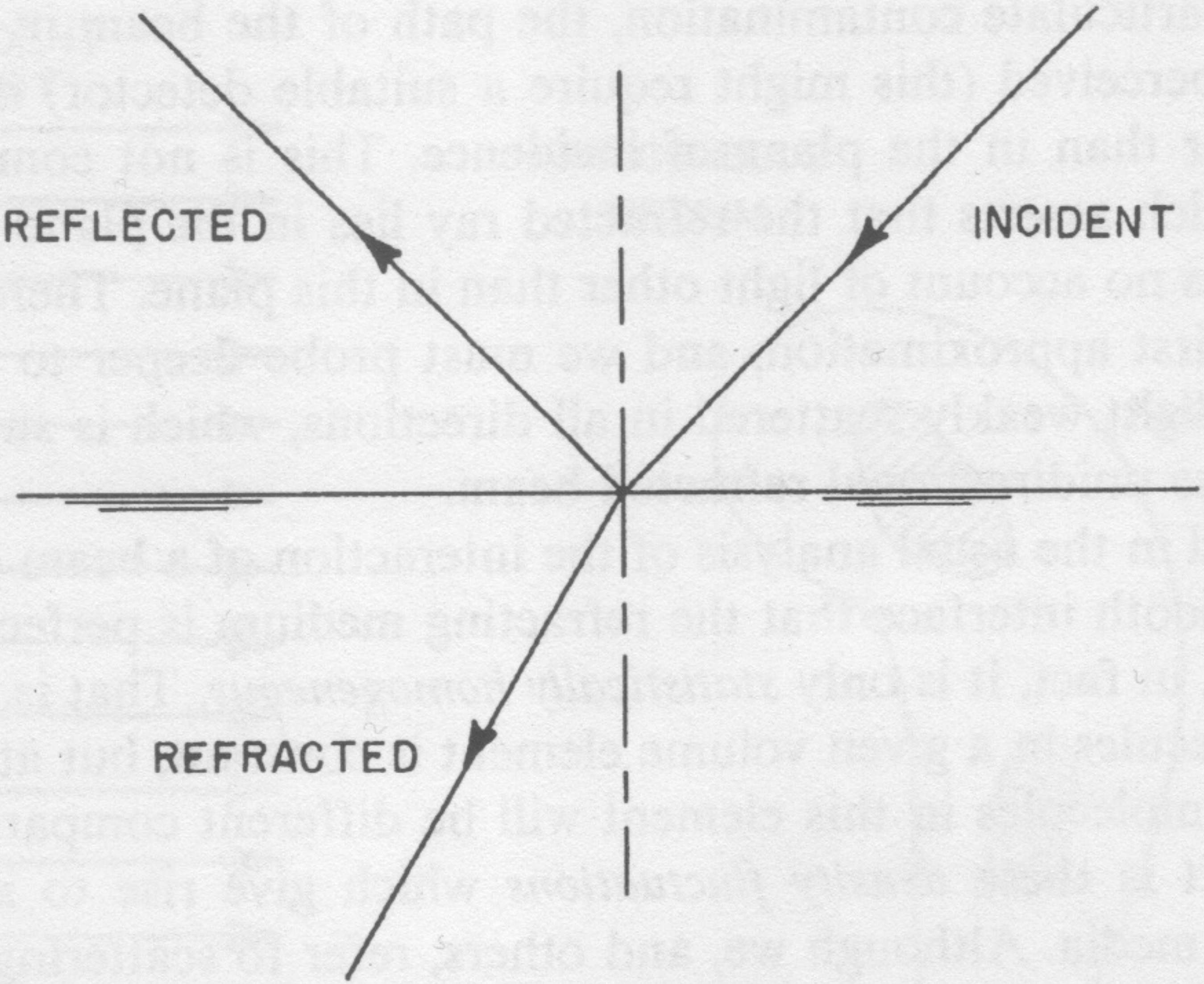


Figure 1.2 Reflection and refraction at an optically smooth interface.

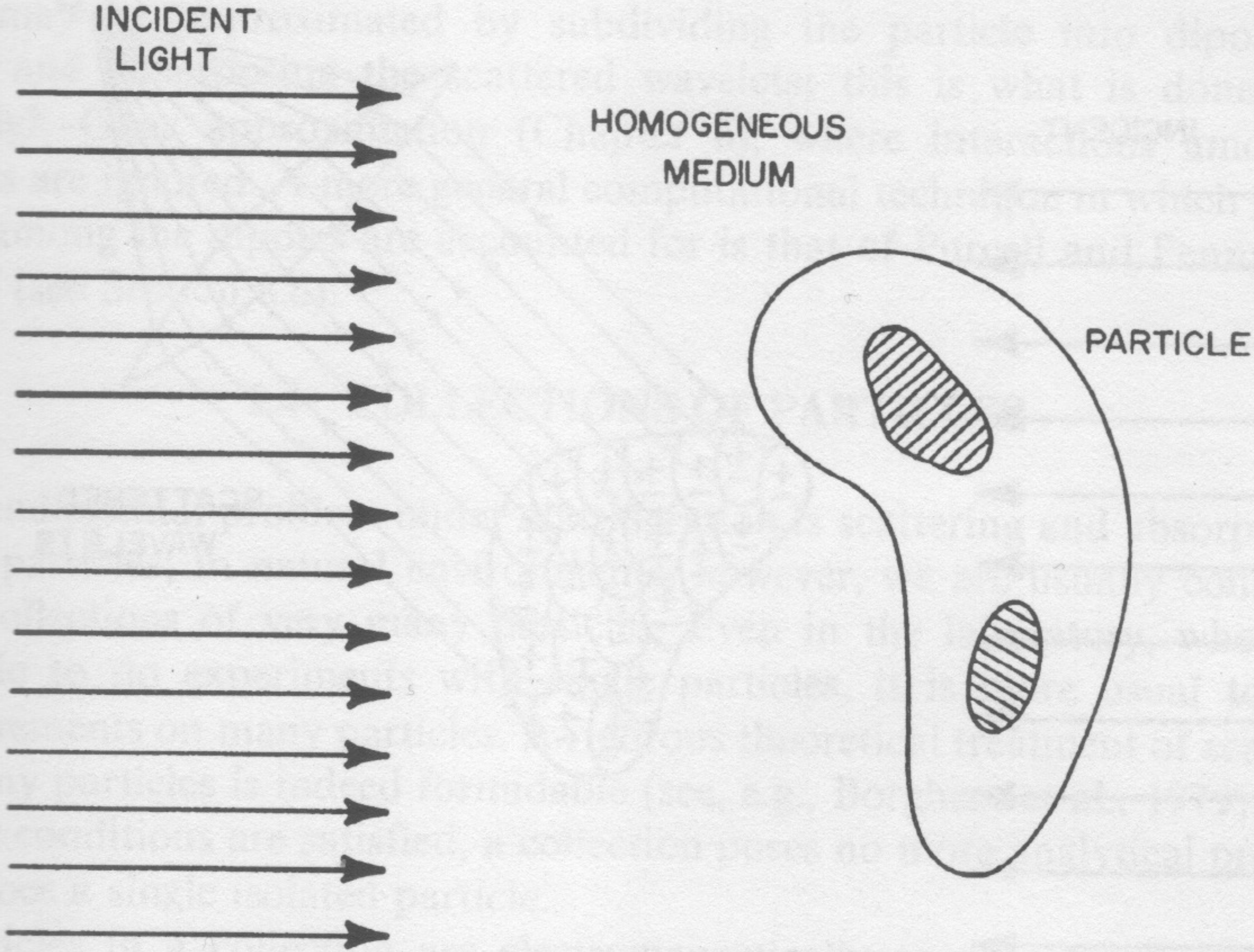


Figure 1.3 Interaction of light with a single particle.

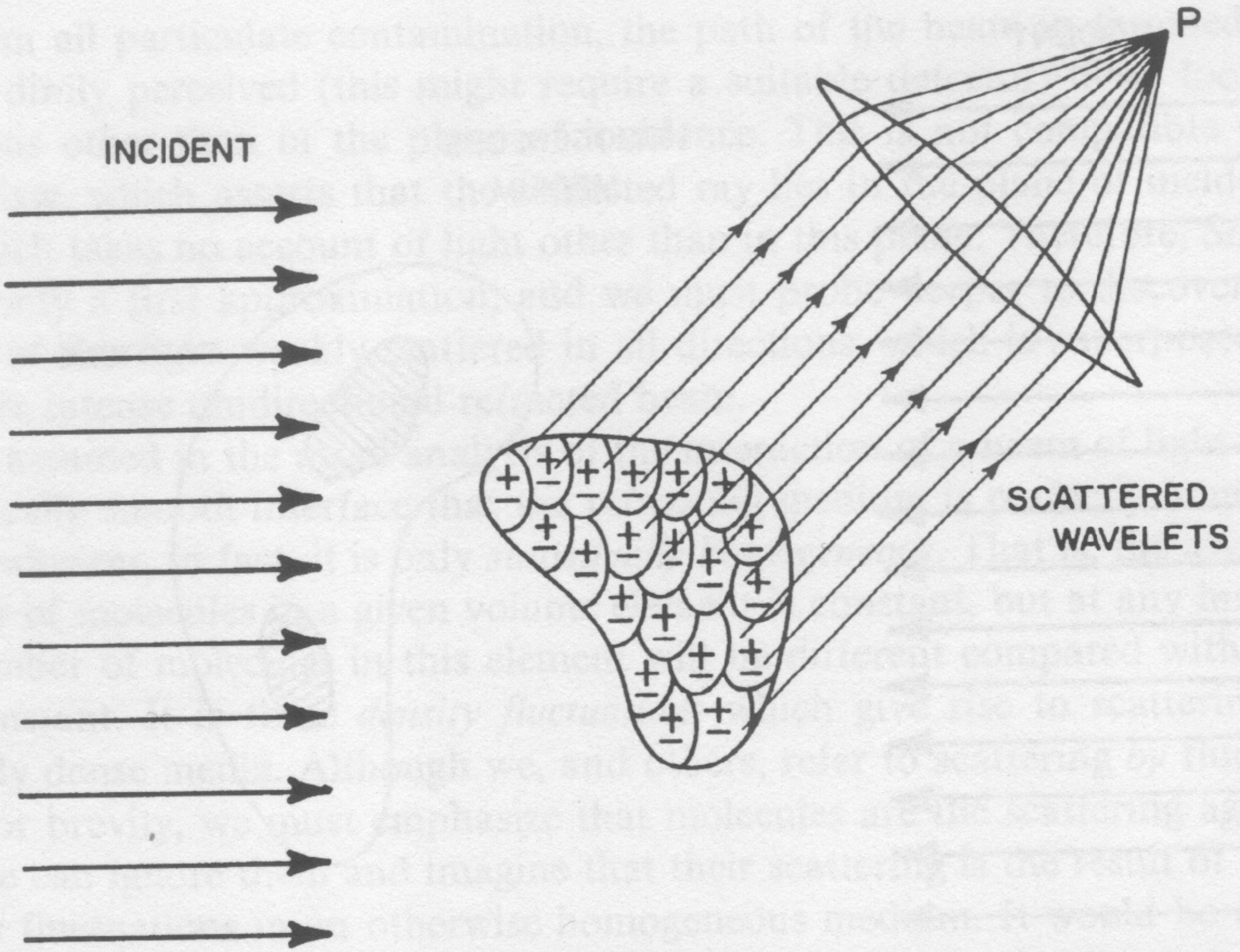


Figure 1.4 The total scattered field at *P* is the resultant of all the wavelets scattered by the regions into which the particle is subdivided.

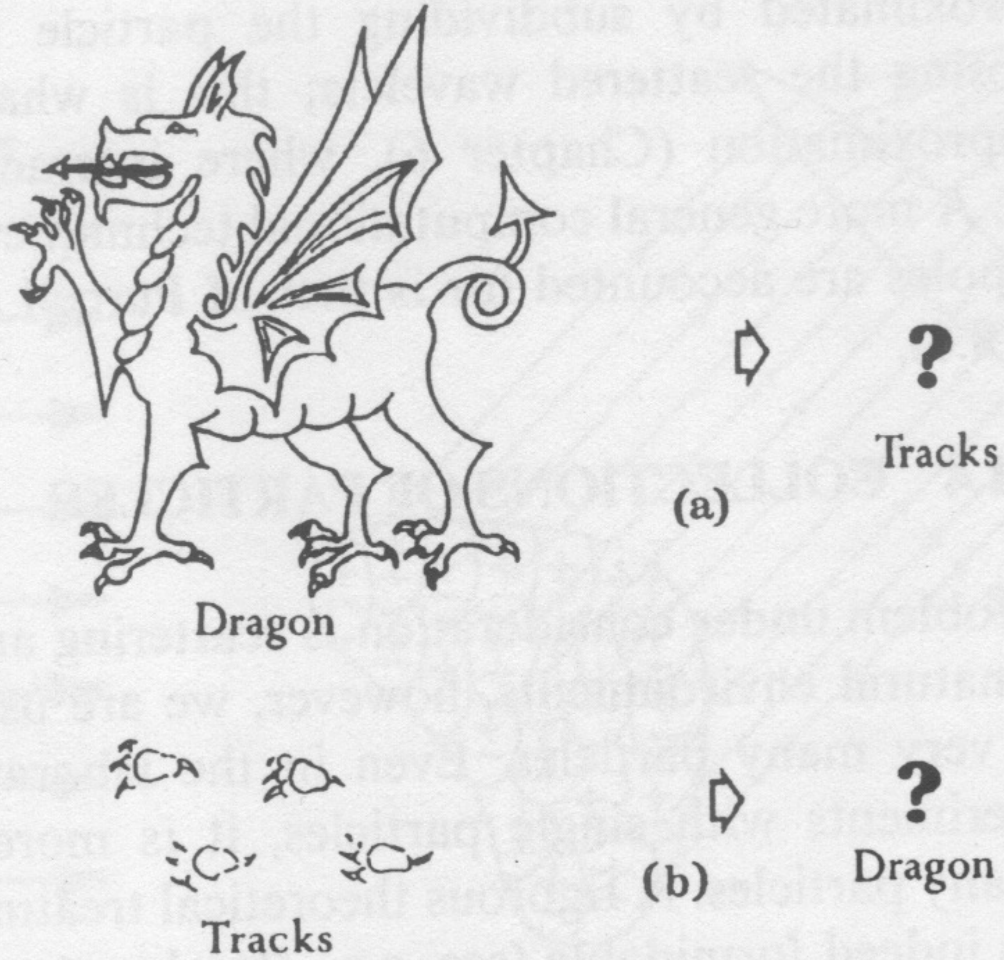


Figure 1.5 (*a*) The direct problem: Describe the tracks of a given dragon. (*b*) The inverse problem: Describe a dragon from its tracks.

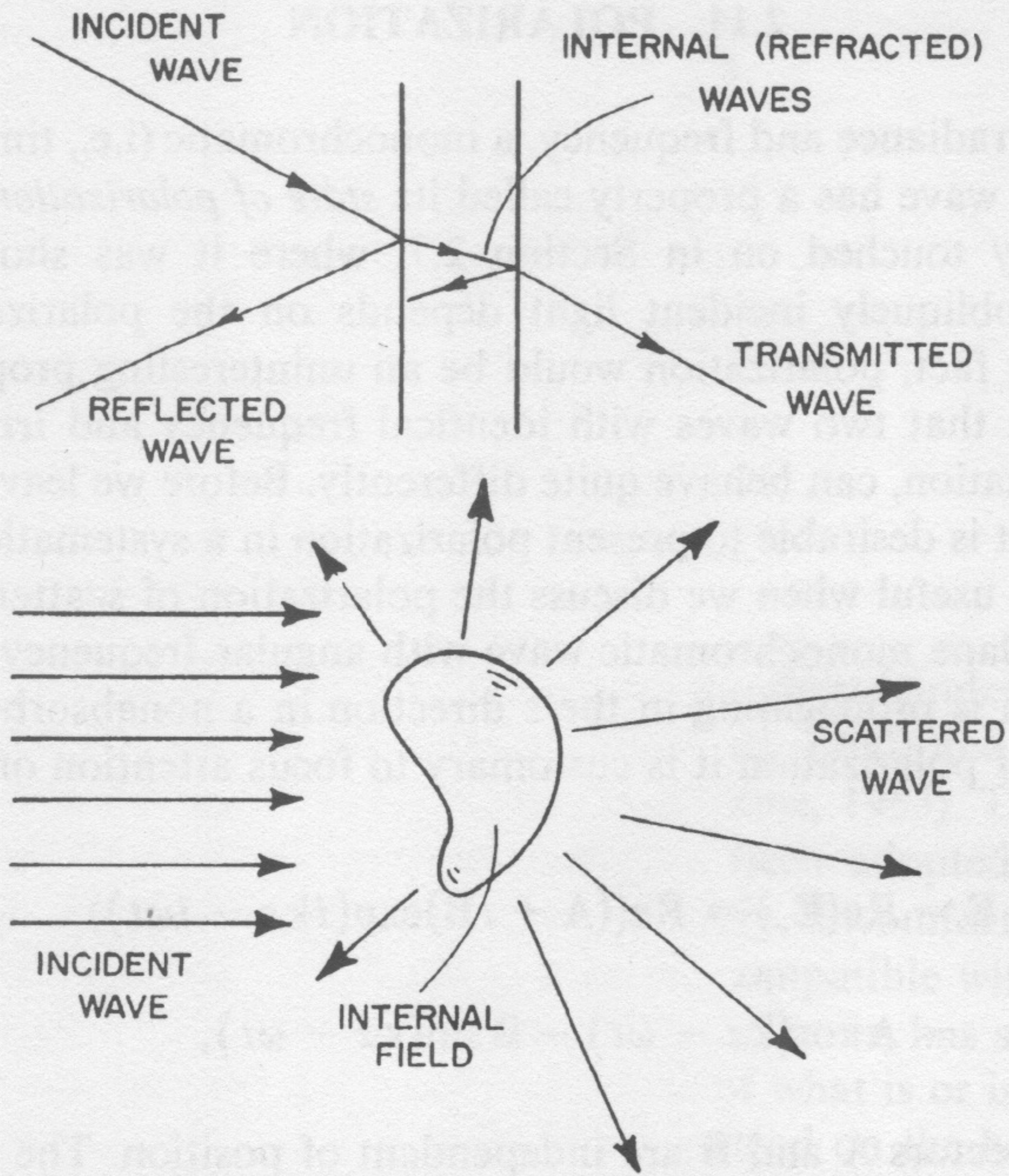


Figure 2.10 Analogy between scattering by a particle and reflection–transmission by a slab.

1 Introduction to scattering theory

Each scattering problem depends on the detailed characteristics of the scattering particle: its size, shape, and refractive index. The size is usually described by the size parameter

$$x = \frac{2\pi a}{\lambda}, \tag{1}$$

where a is a typical radial distance in the particle and and λ is the wavelength of the original electromagnetic field. In the size dependence of scattering, only the ratio a/λ is meaningful. Shape is described by suitable elongation, roughness, or angularity parameters. The constitutive material is characterized by the complex-valued refractive index

$$m = n + \mathrm{i}n', \tag{2}$$

where the real and imaginary parts n and n' are responsible for refraction and absorption of light, respectively. The time dependence of the fields has been chosen to be $\exp(-\mathrm{i}\omega t)$ so that, in physically relevant cases, the imaginary part of the refractive index needs to be non-negative.

1.1 Electromagnetic formulation of the problem

Electromagnetic scattering and absorption is here being assessed from the view point of classical electromagnetics. The natural foundation is provided by the Maxwell equations

$$\begin{aligned} \nabla \cdot \boldsymbol{D} &= \rho, \\ \nabla \times \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t}, \\ \nabla \cdot \boldsymbol{B} &= 0, \\ \nabla \times \boldsymbol{H} &= \boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t}, \end{aligned} \tag{3}$$

where $\boldsymbol{E}$ is the electric field, $\boldsymbol{B}$ is the magnetic flux density, $\boldsymbol{D}$ is the electric displacement, and $\boldsymbol{H}$ is the magnetic field. ρ and $\boldsymbol{j}$ are, respectively, the densities of free charges and currents. In order for the charge and current densities to determine the fields unambiguously, constitutive relations describing the interaction of matter and fields are introduced,

$$\begin{aligned} \boldsymbol{j} &= \sigma \boldsymbol{E}, \\ \boldsymbol{D} &= \epsilon \boldsymbol{E}, \\ \boldsymbol{B} &= \mu \boldsymbol{H}, \end{aligned} \tag{4}$$

where σ is the electric conductivity, ϵ is the electric permittivity, and μ is the magnetic permeability. In what follows, it is assumed that there are no free charges or currents and that the time dependence of the fields is of the harmonic type $\exp(-\mathrm{i}\omega t)$. The Maxwell equations then reduce to the form

$$\begin{aligned} \nabla \cdot \epsilon \boldsymbol{E} &= 0, \\ \nabla \times \boldsymbol{E} &= \mathrm{i}\omega\mu \boldsymbol{B}, \\ \nabla \cdot \boldsymbol{H} &= 0, \\ \nabla \times \boldsymbol{H} &= -\mathrm{i}\omega\epsilon \boldsymbol{E}, \end{aligned} \tag{5}$$

so that the fields $\boldsymbol{E}$ and $\boldsymbol{H}$ fulfil the vector wave equations

$$\begin{aligned} \nabla^2 \boldsymbol{E} + k^2 \boldsymbol{E} &= 0, \\ \nabla^2 \boldsymbol{H} + k^2 \boldsymbol{H} &= 0, \end{aligned} \tag{6}$$

where $k^2 = \omega^2 m^2 / c^2$ and m is the relative refractive index of the scatterer, $m^2 = \epsilon\mu/\epsilon_0\mu_0$.

Denote the internal field of the particle by $(\boldsymbol{E}_1, \boldsymbol{H}_1)$. The external field $(\boldsymbol{E}_2, \boldsymbol{H}_2)$ is the superposition of the original field $(\boldsymbol{E}_i, \boldsymbol{H}_i)$ and the scattered field $(\boldsymbol{E}_s, \boldsymbol{H}_s)$,

$$\begin{aligned} \boldsymbol{E}_2 &= \boldsymbol{E}_i + \boldsymbol{E}_s, \\ \boldsymbol{H}_2 &= \boldsymbol{H}_i + \boldsymbol{H}_s. \end{aligned} \tag{7}$$

In what follows, let us assume that the original field is a plane wave,

$$\begin{aligned} \boldsymbol{E}_i &= \boldsymbol{E}_0 \exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)], \\ \boldsymbol{H}_i &= \boldsymbol{H}_0 \exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)], \boldsymbol{H}_0 = \frac{1}{\omega\mu_0}\boldsymbol{k}\times\boldsymbol{E}_0, \end{aligned} \tag{8}$$

where $\boldsymbol{k}$ is the wave vector of the medium surrounding the particle. Since there are no free currents according to our hypothesis, the tangential components of the fields $\boldsymbol{E}$ and $\boldsymbol{H}$ are continuous across the boundary between the particle and the surrounding medium:

$$\begin{aligned} (\boldsymbol{E}_2 - \boldsymbol{E}_1)\times\boldsymbol{n} &= 0, \\ (\boldsymbol{H}_2 - \boldsymbol{H}_1)\times\boldsymbol{n} &= 0, \end{aligned} \tag{9}$$

at the boundary with an outward normal vector $\boldsymbol{n}$. It is our fundamental goal to solve the Maxwell equations everywhere in space with the boundary conditions given.

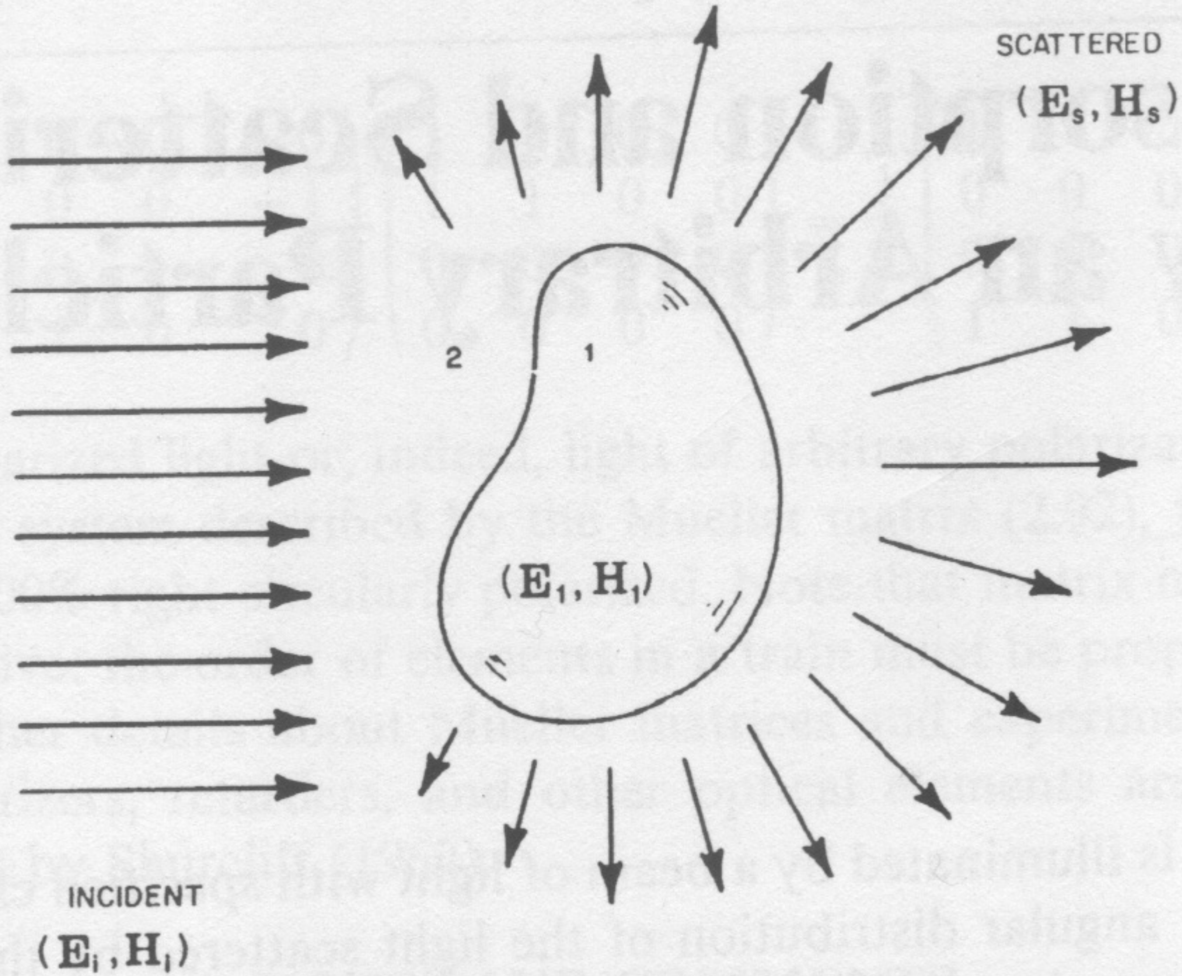


Figure 3.1 The incident field $(\mathbf{E}_i, \mathbf{H}_i)$ gives rise to a field $(\mathbf{E}_1, \mathbf{H}_1)$ inside the particle and a scattered field $(\mathbf{E}_s, \mathbf{H}_s)$ in the medium surrounding the particle.

1.2 Amplitude scattering matrix

Let us place an arbitrary particle in a plane wave field according to the figure (cf. Bohren & Huffman). The propagation directions of the original and scattered fields $\boldsymbol{e}_z$ and $\boldsymbol{e}_r$ define a scattering plane, and the original field is divided into components perpendicular and parallel to that plane,

$$\boldsymbol{E}_i = (E_{0\perp}\boldsymbol{e}_{i\perp} + E_{0\|}\boldsymbol{e}_{i\|}) \exp[\mathrm{i}(kz - \omega t)] = E_{i\perp}\boldsymbol{e}_{i\perp} + E_{i\|}\boldsymbol{e}_{i\|}. \tag{10}$$

In the radiation zone, that is, far away from the scattering particle, the scattered field is a transverse spherical wave (cf. Jackson),

$$\boldsymbol{E}_s = \frac{\exp(\mathrm{i}kr)}{-\mathrm{i}kr}\boldsymbol{A}, \boldsymbol{e}_r \cdot \boldsymbol{A} = 0, \tag{11}$$

so that

$$\boldsymbol{E}_s = E_{s\perp}\boldsymbol{e}_{s\perp} + E_{s\|}\boldsymbol{e}_{s\|}, \tag{12}$$

where

$$\begin{aligned} \boldsymbol{e}_{s\perp} &= -\boldsymbol{e}_{\phi}, \\ \boldsymbol{e}_{s\|} &= \boldsymbol{e}_{\theta}. \end{aligned} \tag{13}$$

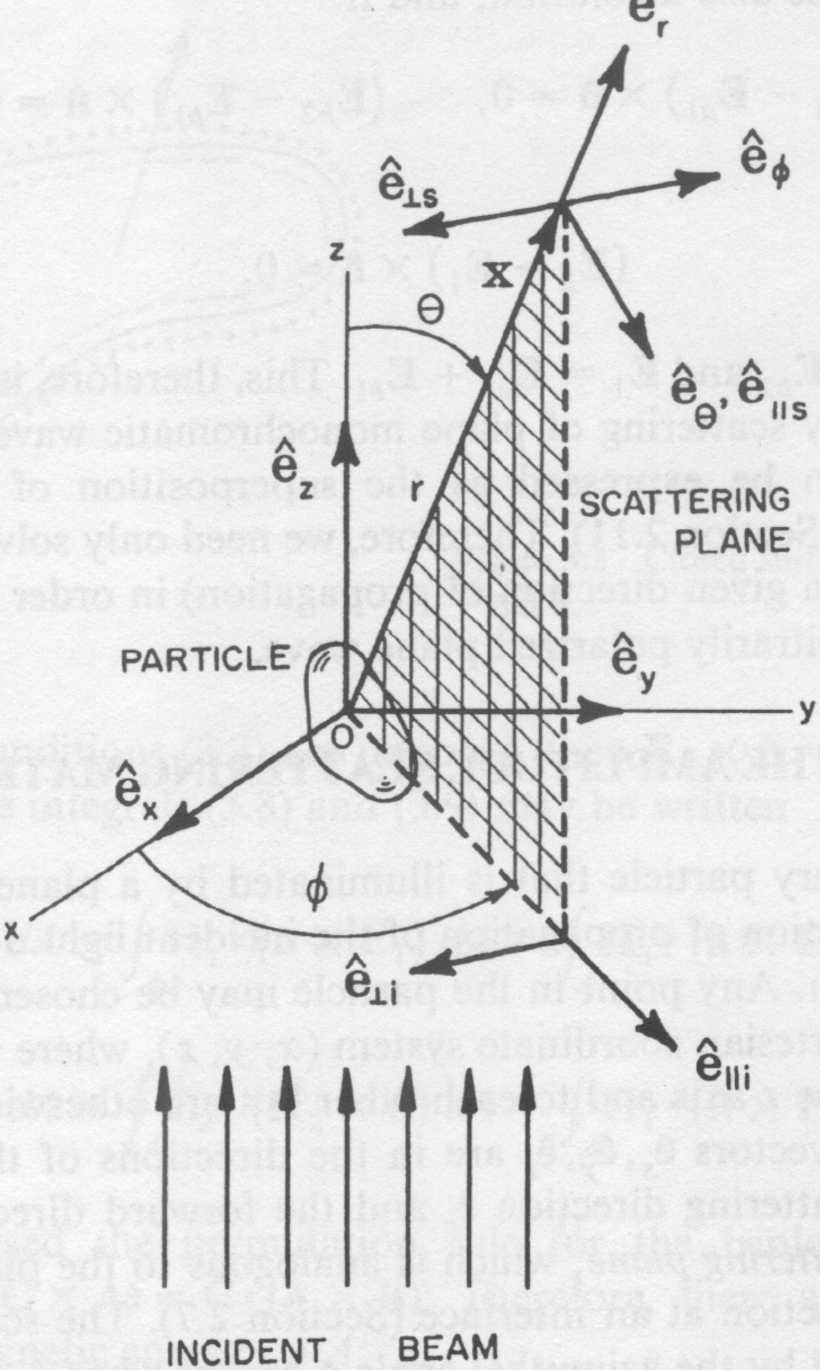


Figure 3.3 Scattering by an arbitrary particle.

Due to the linearity of the boundary conditions, the amplitude of the scattered field depends linearly on the amplitude of the original field. In a matrix form,

$$\begin{bmatrix} E_{s\perp} \\ E_{s\|} \end{bmatrix} = \frac{\exp[\mathrm{i}(kr - kz)]}{-\mathrm{i}kr} \begin{bmatrix} S_1 & S_4 \\ S_3 & S_2 \end{bmatrix} \begin{bmatrix} E_{i\perp} \\ E_{i\|} \end{bmatrix}, \tag{14}$$

where the complex-valued amplitude-scattering-matrix elements S_j $(j = 1, 2, 3, 4)$ generally depend on the scattering angle θ and the azimuthal angle ϕ. Since only the relative phases are important, the amplitude scattering matrix has seven free parameters.

1.3 Stokes parameters and scattering matrix

In the medium surrounding the particle, the time-averaged Poynting vector $\boldsymbol{S}$ can be divided into the Poynting vectors of the original field, scattered field, and that showing the interaction of the original and scattered fields,

$$\boldsymbol{S} = \frac{1}{2}\mathrm{Re}(\boldsymbol{E}_2 \times \boldsymbol{H}_2^*) = \boldsymbol{S}_i + \boldsymbol{S}_s + \boldsymbol{S}_e, \tag{15}$$

where

$$\begin{aligned} \boldsymbol{S}_i &= \frac{1}{2}\mathrm{Re}(\boldsymbol{E}_i \times \boldsymbol{H}_i^*), \\ \boldsymbol{S}_s &= \frac{1}{2}\mathrm{Re}(\boldsymbol{E}_s \times \boldsymbol{H}_s^*), \\ \boldsymbol{S}_e &= \frac{1}{2}\mathrm{Re}(\boldsymbol{E}_i \times \boldsymbol{H}_s^* + \boldsymbol{E}_s \times \boldsymbol{H}_i^*). \end{aligned} \tag{16}$$

In the radiation zone, the power incident on a surface element ΔA perpendicular to the radial direction is

$$\boldsymbol{S}_s \cdot \boldsymbol{e}_r = \frac{k}{2\omega\mu}\frac{|\boldsymbol{A}|^2}{k^2}\Delta\Omega, \Delta\Omega = \frac{\Delta A}{r^2} \tag{17}$$

and $|\boldsymbol{A}|^2$ can be measured as a function of angles. By placing polarizers in between the scattering particle and the detector, we can measure the Stokes parameters of the scattered field (Bohren & Huffman),

$$\begin{aligned} I_s &= \langle |E_{s\perp}|^2 + |E_{s\parallel}|^2 \rangle, \\ Q_s &= \langle -|E_{s\perp}|^2 + |E_{s\parallel}|^2 \rangle, \\ U_s &= 2\mathrm{Re}E_{s\perp}^* E_{s\parallel}, \\ V_s &= -2\mathrm{Im}E_{s\perp}^* E_{s\parallel}. \end{aligned} \tag{18}$$

Thus, I_s gives the scattered intensity, Q_s gives the difference between the intensities in the scattering plane and perpendicular to the scattering plane, U_s gives the difference between $+\pi/4$ and $-\pi/4$ -polarized intensities and, lastly, V_s gives the difference between right-handed and left-handed circularly polarized intensities. The factor $k/2\omega\mu_0$ has been omitted from the intensities; it is not needed since, in practice, relative intensities are measured instead of absolute ones. The Stokes parameters fully describe the polarization state of an electromagnetic field.

The scattering matrix S interrelates the Stokes parameters of the original field and the scattered field, and can be derived from the amplitude scattering matrix:

$$\boldsymbol{I}_s = \frac{1}{k^2r^2}S\boldsymbol{I}_i, \tag{19}$$

where the Stokes vectors

$$\begin{aligned}\boldsymbol{I}_s &= (I_s, Q_s, U_s, V_s)^T,\\ \boldsymbol{I}_i &= (I_i, Q_i, U_i, V_i)^T.\end{aligned} \tag{20}$$

The information about the angular dependence of scattering is fully contained in the 16 elements of the scattering matrix. For a single scattering particle, it has seven free parameters whereas, for an ensemble of particles, all 16 elements can be free. Symmetries reduce the number of free parameters: for example, for a spherical particle, there are three free parameters.

$$\begin{pmatrix} I_s \\ Q_s \\ U_s \\ V_s \end{pmatrix} = \frac{1}{k^2 r^2} \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{pmatrix} \begin{pmatrix} I_i \\ Q_i \\ U_i \\ V_i \end{pmatrix} \tag{3.16}$$

$$S_{11} = \tfrac{1}{2}\left(|S_1|^2 + |S_2|^2 + |S_3|^2 + |S_4|^2\right),$$

$$S_{12} = \tfrac{1}{2}\left(|S_2|^2 - |S_1|^2 + |S_4|^2 - |S_3|^2\right),$$

$$S_{13} = \mathrm{Re}\{S_2 S_3^* + S_1 S_4^*\},$$

$$S_{14} = \mathrm{Im}\{S_2 S_3^* - S_1 S_4^*\},$$

$$S_{21} = \tfrac{1}{2}\left(|S_2|^2 - |S_1|^2 - |S_4|^2 + |S_3|^2\right),$$

$$S_{22} = \tfrac{1}{2}\left(|S_2|^2 + |S_1|^2 - |S_4|^2 - |S_3|^2\right),$$

$$S_{23} = \mathrm{Re}\{S_2 S_3^* - S_1 S_4^*\},$$

$$S_{24} = \mathrm{Im}\{S_2 S_3^* + S_1 S_4^*\},$$

$$S_{31} = \mathrm{Re}\{S_2 S_4^* + S_1 S_3^*\},$$

$$S_{32} = \mathrm{Re}\{S_2 S_4^* - S_1 S_3^*\},$$

$$S_{33} = \mathrm{Re}\{S_1 S_2^* + S_3 S_4^*\},$$

$$S_{34} = \mathrm{Im}\{S_2 S_1^* + S_4 S_3^*\},$$

$$S_{41} = \mathrm{Im}\{S_2^* S_4 + S_3^* S_1\},$$

$$S_{42} = \mathrm{Im}\{S_2^* S_4 - S_3^* S_1\},$$

$$S_{43} = \mathrm{Im}\{S_1 S_2^* - S_3 S_4^*\},$$

$$S_{44} = \mathrm{Re}\{S_1 S_2^* - S_3 S_4^*\}.$$

For an unpolarized incident field, the Stokes parameters of the scattered field are

$$\begin{aligned} I_s &= \frac{1}{k^2r^2} S_{11} I_i, \\ Q_s &= \frac{1}{k^2r^2} S_{21} I_i, \\ U_s &= \frac{1}{k^2r^2} S_{31} I_i, \\ V_s &= \frac{1}{k^2r^2} S_{41} I_i. \end{aligned} \quad (21)$$

Thus, S_{11} gives the angular distribution of scattered intensity and the total degree of polarization is

$$P_{\text{tot}} = \frac{\sqrt{S_{21}^2 + S_{31}^2 + S_{41}^2}}{S_{11}}. \quad (22)$$

Scattering polarizes light.

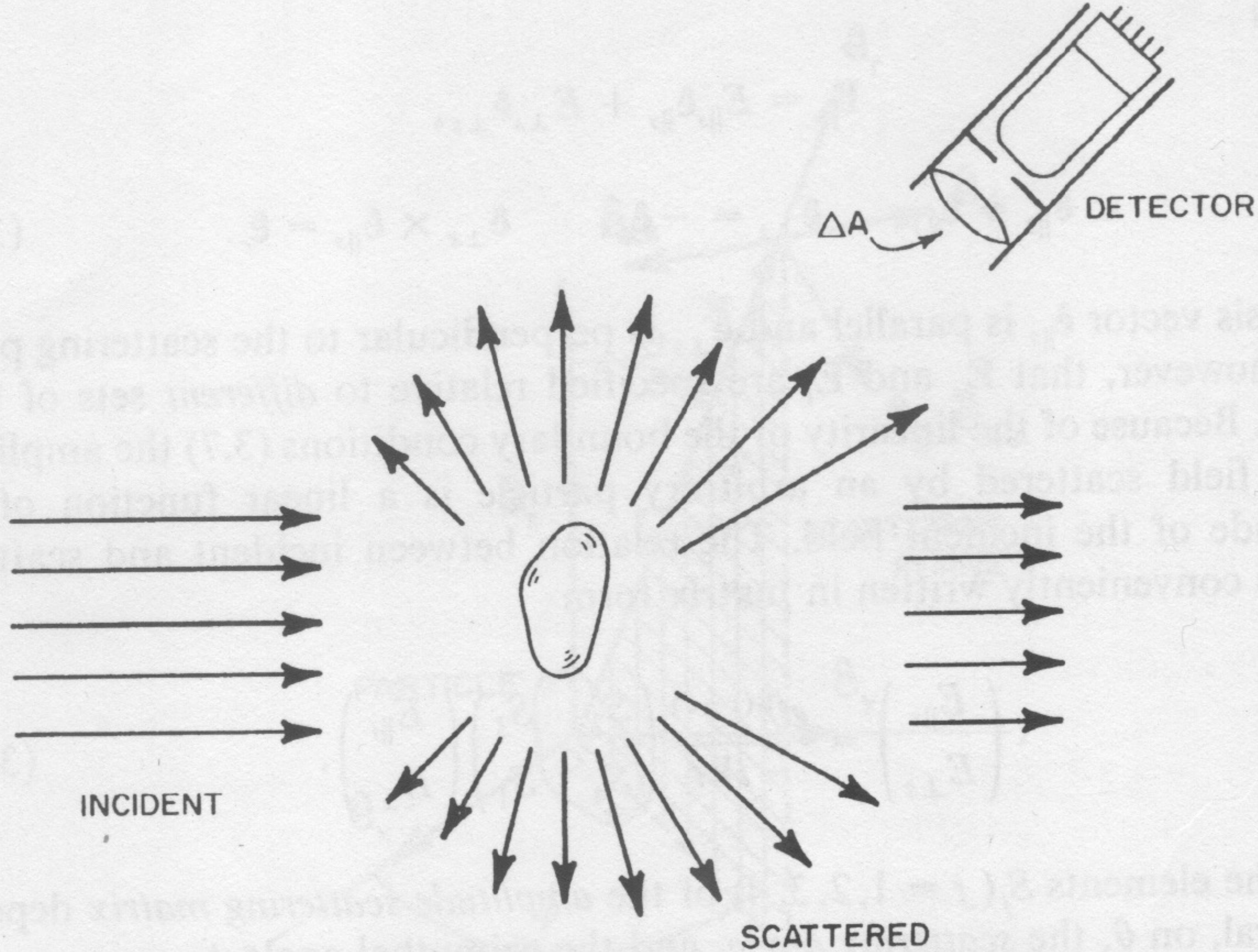


Figure 3.4 The collimated detector responds only to the scattered light.