Electromagnetic Scattering I (53919, 5 cr)
Exercise 4

1. Fraunhofer diffraction by a spherical particle ($x = 2\pi a/\lambda$, where a is the radius and λ is the wavelength) is

$$D(x,\theta) = x^2 \cos\theta \left[\frac{2J_1(x\sin\theta)}{x\sin\theta}\right]^2 \Theta(90^\circ - \theta) + J_0(x)^2 + J_1(x)^2,$$

where θ is the scattering angle, J_1 is a Bessel function of the first kind and of the order 1, and Θ is the Heaviside step function,

$$\Theta(s) = 1, s \geq 0$$
$$\Theta(s) = 0, s < 0.$$

Show that

$$\int_{(4\pi)} \frac{d\Omega}{4\pi} D(x,\theta) = 1. \quad (1)$$

What is your interpretation of the term $D(x,\theta)/(4\pi)$?

The following relationships are valid for the Bessel functions:

$$J_0'(y) = -J_1(y)$$
$$J_{n-1}(y) = \frac{n}{y} J_n(y) + J_n'(y)$$

(6 p)

2. Consider diffraction by a circular hole in Smythe–Kirchhoff approximation for normal incidence. Show that the ratio of the transmitted power to incident power (i.e., the transmission coefficient) is

$$T = 1 - \frac{1}{2x}\int_0^{2x} dt\ J_0(t),$$

where $x = ka$ (a is the radius of the hole).

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3. In the anomalous diffraction approximation (ADA), the size of the scatterer is assumed much larger than the wavelength (λ, wave number $k = 2\pi/\lambda$), and the refractive index (m) of the scatterer should not deviate much from that of the surrounding medium.

In ADA, the extinction and absorption cross sections are integrals over the 2D projection $S_\perp$ of the particle,

$$\sigma_{\rm e} = 2\int_{S_\perp} d^2\underline{r}_\perp \mathrm{Re}\left\{1 - \exp\left[-i\Phi(\underline{r}_\perp)\right]\right\},$$
$$\sigma_{\rm a} = \int_{S_\perp} d^2\underline{r}_\perp \left(1 - \exp\left\{2\mathrm{Im}\left[\Phi(\underline{r}_\perp)\right]\right\}\right),$$

(scattering cross section $\sigma_{\rm s} = \sigma_{\rm e} - \sigma_{\rm a}$) where the phase Φ is a function of the complex refractive index m, the wave number k, and the distance d a directly transmitted ray travels inside the scatterer,

$$\Phi(\underline{r}_\perp) = (m-1)kd(\underline{r}_\perp).$$

The scattering-matrix element $S_{11} = k^2 \sigma_{\mathrm{s}} P_{11}/4\pi$ is

$$S_{11}(\theta) \quad = \quad \frac{k^4}{4\pi^2} \left| \int_{S_\perp} d^2\underline{\mathrm{r}}_\perp \exp(ikx\theta) \left\{1 - \exp\left[i\Phi(\underline{\mathrm{r}}_\perp)\right]\right\} \right|^2,$$

when xz-plane is the scattering plane.

Calculate σ_{e}, σ_{a}, σ_{s}, and S_{11} for a spherical scatterer of radius a in ADA.

(18 p)