Electromagnetic Scattering I (53919, 5 cr)
Exercise 3

1. Derive Rayleigh scattering matrix from Lorenz-Mie scattering matrix (remember that, in Rayleigh scattering, $x \ll 1$; see Bohren & Huffman). The amplitude scattering matrix elements of Lorenz-Mie scattering matrix are

$$\begin{aligned} S_1 &= \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)}(a_n \pi_n + b_n \tau_n), \\ S_2 &= \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)}(a_n \tau_n + b_n \pi_n), \end{aligned}$$

where

$$\begin{aligned} a_n &= \frac{m\psi_n(mx)\psi_n'(x) - \psi_n(x)\psi_n'(mx)}{m\psi_n(mx)\xi_n'(x) - \xi_n(x)\psi_n'(mx)}, \\ b_n &= \frac{\psi_n(mx)\psi_n'(x) - m\psi_n(x)\psi_n'(mx)}{\psi_n(mx)\xi_n'(x) - m\xi_n(x)\psi_n'(mx)} \end{aligned}$$

and the special functions are

$$\begin{aligned} \pi_n &= \frac{2n-1}{n-1}\cos\theta \pi_{n-1} - \frac{n}{n-1}\pi_{n-2}, \\ \tau_n &= n\cos\theta\pi_n - (n+1)\pi_{n-1}, \\ \pi_0 &= 0, \\ \pi_1 &= 1, \end{aligned}$$

$$\begin{aligned} \psi_n(\rho) &= \rho j_n(\rho), \\ \xi_n(\rho) &= \rho h_n^{(1)}(\rho) = \rho[j_n(\rho) + i n_n(\rho)], \\ j_n(\rho) &= \frac{\rho^n}{(1\cdot 3\cdot 5\cdot \ldots \cdot(2n+1)}\left[1 - \frac{\frac{1}{2}\rho^2}{1!(2n+3)} + \frac{(\frac{1}{2}\rho^2)^2}{2!(2n+3)(2n+5)} - \ldots\right], \\ n_n(\rho) &= -\frac{1\cdot 3\cdot 5\cdot \ldots \cdot(2n-1)}{\rho^{(n+1)}}\left[1 - \frac{\frac{1}{2}\rho^2}{1!(1-2n)} + \frac{(\frac{1}{2}\rho^2)^2}{2!(1-2n)(3-2n)} - \ldots\right]. \end{aligned}$$

(12 points)

1. A vector spherical harmonics expansion for the electromagnetic field scattered by a sphere is presented in the book *Classical Electrodynamics* by J. D. Jackson (Chapter 16.9 in the 2nd edition; see also Lecture notes 9). Using this expression for the scattered field, derive the amplitude scattering matrix for a spherical particle.

(18 points)