Electromagnetic Scattering I (53919, 5 cr)
Exercise 2

1. Based on the results of Problem 4 in Exercise 1, derive the scattered electromagnetic field in the radiation zone (far zone), and all the elements of the Rayleigh scattering matrix. The relation between the scattering matrix and amplitude scattering matrix elements is presented, e.g., in Bohren & Huffman. (12 points)

2. A plane wave is scattered by two small interacting spherical particles of radius $a << \lambda$, where λ is the wavelength of the incident plane wave (size parameter $x = 2\pi a/\lambda$). One sphere is set to the origin and the location of the other sphere is denoted by a vector $\mathbf{d}$. In the dipole approximation, the internal fields of the particles $\mathbf{E}_1$ and $\mathbf{E}_2$ are related through

$$
\begin{aligned}
\mathbf{E}_1 &= \mathbf{E}_{i1} + \beta \bar{\mathbf{T}}(u, v) \cdot \mathbf{E}_2 \\
\mathbf{E}_2 &= \mathbf{E}_{i2} + \beta \bar{\mathbf{T}}(u, v) \cdot \mathbf{E}_1,
\end{aligned}
$$

where $\mathbf{E}_{i1}$ and $\mathbf{E}_{i2}$ are the incident fields at the locations of the particles, and the polarizability (m is the refractive index)

$$
\beta = x^3 \frac{m^2 - 1}{m^2 + 2}.
$$

Transformation $\bar{\mathbf{T}}$ denotes the interaction between the particles:

$$
\begin{aligned}
\bar{\mathbf{T}}(u, v) &= u\bar{\mathbf{I}} + v\mathbf{d}\mathbf{d}/d^2 \\
u &= \mathrm{e}^{i\rho}(\rho^2 + i\rho - 1)/\rho^3 \\
v &= \mathrm{e}^{i\rho}(-\rho^2 - i3\rho + 3)/\rho^3, \;\; \rho = kd.
\end{aligned}
$$

Solve the electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$. (Muinonen 1990, PhD thesis)

Note the following rules for an operator $\mathbf{ab}$:
$(\mathbf{ab}) \cdot \mathbf{c} = \mathbf{a}(\mathbf{b} \cdot \mathbf{c})$
$\mathbf{c} \cdot (\mathbf{ab}) = (\mathbf{c} \cdot \mathbf{a})\mathbf{b}$
The unit operator $\bar{\mathbf{I}}$ has no effect on the operator $\mathbf{ab}$ or the vector $\mathbf{c}$.

(12 points)