5th exercises for SIM'2018

Ex. 1

Fit nonlinear model to observed degree of linear polarization of comet Hale-Bopp in red wavelength filter. Download data from course webpage (`dataRed.dat`). Use trigonometric model

$$y_i = \mathrm{f}_i(\boldsymbol{\beta}) = \beta_1 \, \sin(x_i)^{\beta_2} \, \cos(x_i/2)^{\beta_3} \, \sin(x_i - \beta_4). \tag{1}$$

a) Program function $S(b_1, b_2, b_3, b_4) = \sum^n (y_i - \mathrm{f}_i(b_1, b_2, b_3, b_4))^2$ in your computing environment. Use minimization procedure to find best estimates b_1, b_2, b_3, b_4. (If you cannot use minimization, test few choices for parameters yourself and choose the best ones).

b) Plot data together with the best fit function.

c) Compute sum of squared residuals, SSE, and residual variance s^2.

d) Compute test statistics $t_i = \frac{b_i}{s\sqrt{m^{ii}}}$ for the parameters. For $m^{ii} = [M^{-1}]_{ii}$ you need the matrix $\mathbf{F}(\boldsymbol{b})$ as in Eq. (4.15). Partial derivatives of f are given in the end of this page.

e) Which parameter is the most uncertain, i.e. has smallest value of test statistics? Test it's p-value for the hypothesis H_0 that it could be removed from the model.

Ex. 2

Do kernel density estimation for one-dimensional data `asteroid_density.dat`, where the densities (in g/cm^3) of some asteroids are recorded. Test either few different kernels or few values of smoothing parameter h. Plot the density estimates. Can there be 'unphysical' features in the density estimate?

$$\frac{\partial \mathrm{f}}{\partial b_1} = -\sin(x)^{b_2} \, \cos\left(\frac{x}{2}\right)^{b_3} \, \sin(b_4 - x) \tag{2}$$

$$\frac{\partial \mathrm{f}}{\partial b_2} = -b_1 \, \sin(x)^{b_2} \, \cos\left(\frac{x}{2}\right)^{b_3} \, \sin(b_4 - x) \, \log\left(\sin(x)\right) \tag{3}$$

$$\frac{\partial \mathrm{f}}{\partial b_3} = -b_1 \, \sin(x)^{b_2} \, \cos\left(\frac{x}{2}\right)^{b_3} \, \sin(b_4 - x) \, \log\left(\cos\left(\frac{x}{2}\right)\right) \tag{4}$$

$$\frac{\partial \mathrm{f}}{\partial b_4} = -b_1 \, \sin(x)^{b_2} \, \cos\left(\frac{x}{2}\right)^{b_3} \, \cos(b_4 - x) \tag{5}$$