Galaxies and cosmology equation sheet

- Hubble's law: $v_r = H_0 d + v_{\rm pec}$
- Schechter function: $\phi(L)dL = \phi^* \left(\frac{L}{L^*}\right)^\alpha e^{-(L/L^*)} \frac{dL}{L^*}$
- de Vaucouleurs profile: $\log(I/I_e) = -3.33[(r/r_e)^{1/4} - 1]$
- Sérsic profile: $\log(I/I_e) = k[(r/r_e)^{1/n} - 1]$
- Disc galaxy profile: $I(R) = I_0 e^{-R/R_d}, \quad I_0 = \frac{L}{2\pi R_d^2}$
- Poisson equation: $\nabla^2\Phi = 4\pi G\rho$
- Virial theorem: $2\langle E_k\rangle + \langle E_p\rangle + \langle \sum_\alpha \mathbf{F}^\alpha_{\rm ext} \cdot \mathbf{x}_\alpha \rangle = 0$
- Timescale for strong encounters: $t_{\rm s} = \frac{V^3}{4\pi G^2 m^2 n}$
- Relaxation timescale: $t_{\rm relax} = \frac{V^3}{8\pi G^2 m^2 n \ln\Lambda} = \frac{t_{\rm s}}{2\ln\Lambda}$
- Equation of continuity: $\frac{\partial n}{\partial t} + \frac{\partial(nv)}{\partial x} = 0$
- Boltzmann equation: $\frac{\partial f(\mathbf{x},\mathbf{v},t)}{\partial t} + \mathbf{v}\cdot\nabla f - \nabla\Phi \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$
- Robertson-Walker metric: $ds^2 = c^2dt^2 - dl^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\vartheta^2 + \sin^2\vartheta d\phi^2)\right]$
- Conformal time: $\tau(t) = \int_0^t \frac{cdt'}{a(t')}$
- Redshift and scale factor: $1 + z = \frac{\lambda_0}{\lambda_e} = \frac{a(t_0)}{a(t_e)}$
- Angular diameter distance:$d_A = \frac{a_0 r}{1+z}$
  Luminosity distance: $d_L = a_0 r(1+z) = d_A(1+z)^2$
- Einstein's field equation: $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu}$
- Friedmann equation I: $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{P}{c^2}\right) + \frac{\Lambda c^2}{3}$
  Friedmann equation II: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda c^2}{3}$
- Critical density: $\rho_{\rm crit} = \frac{3H^2(t)}{8\pi G}$
- Hubble constant: $H(z) = \left(\frac{\dot{a}}{a}\right)(z) = H_0 E(z)$
  $E(z) = [\Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + (1-\Omega_0)(1+z)^2 + \Omega_{\Lambda,0}]^{1/2}$
- Age of the Universe: $t(z) = \int_0^{a(z)} \frac{da}{\dot{a}} = \frac{1}{H_0}\int_z^\infty \frac{dz}{(1+z)E(z)}$
- Angular diameter distance in comoving coordinates: $r = f_K\left[\frac{c}{H_0 a_0}\int_0^z \frac{dz}{E(z)}\right]$

  $f_K(\chi) = \sin\chi \ (K=+1); \ f_K(\chi) = \chi \ (K=0); \ f_K(\chi) = \sinh\chi \ (K=-1)$
- Mattig's formula: $a_0 r = \frac{2c}{H_0}\frac{\Omega_0 z + (2-\Omega_0)[1-(\Omega_0 z+1)^{1/2}]}{\Omega_0^2(1+z)}$

- Photometric fine structure of elliptical galaxies: $\Delta r(t) = \sum_{k\geq 3}(a_k \cos kt + b_k \sin kt)$
- Isotropic elliptical galaxy: $\left(\frac{V_{\rm rot}}{\sigma}\right) \approx \sqrt{\epsilon/(1-\epsilon)}$
- Fundamental relation: $R_e \propto \sigma^{1.24} I_e^{-0.82}$
- Spiralequation: $\frac{1}{\tan i} = R\left|\frac{\partial \phi}{\partial R}\right| = R\left|\frac{\partial f}{\partial R}\right|$
- Toomre Q-parameter: $Q = \frac{\kappa \sigma_R}{3.36 G\Sigma} \lesssim 1$
- Eddington luminosity: $L_{\rm edd} = \frac{4\pi G M m_p c}{\sigma_T}$
- Superluminar motion: $V_{\rm obs} = \frac{V \sin\theta}{1 - V/c \cos\theta}$
- Jacobi radius: $x = \pm r_J$, $r_J = D\left[\frac{m}{3M+m}\right]^{1/3}$
- Metallicity of a closed box: $Z(t) = Z_0 + p \ln\left[\frac{M_g(t=0)}{M_g(t)}\right]$
- Dynamical friction: $-\frac{dV}{dt} = \frac{4\pi G^2(M+m)}{V^2} nm \ln\Lambda$
- Two-point correlation function and power spectrum:
  $dN(r) = N_0[1+\xi(r)]dV$
  $\xi(r) = \frac{V}{2\pi^2}\int |\Delta_k|^2 \frac{\sin kr}{kr} k^2 dk = \frac{V}{2\pi^2}\int P(k)\frac{\sin kr}{kr} k^2 dk$
  $P(k) = \frac{1}{V}\int_0^\infty \xi(r)\frac{\sin kr}{kr} 4\pi r^2 dr$
- Inflation: $\rho + 3P/c^2 < 0 \Rightarrow a \propto e^{Ht}, H = \sqrt{8\pi G\rho_{\rm vac}/3}$
- The early Universe: $t = \left(\frac{3c^2}{32\pi G a_B T^4}\right)^{1/2} \approx 230\ {\rm s}\left(\frac{10^9\ {\rm K}}{T}\right)^2$
- Gas dynamics: Equation of continuity: $\frac{d\rho}{dt} = -\rho\nabla\cdot\vec{v}$
  Equation of motion (Euler equation): $\frac{d\vec{v}}{dt} = -\frac{1}{\rho}\nabla p - \nabla\phi$
  Gravity equation (Poisson equation): $\nabla^2\phi = 4\pi G\rho$
- Growth of small density perturbations: $\frac{d^2\Delta}{dt^2} + 2\left(\frac{\dot a}{a}\right)\frac{d\Delta}{dt} = \Delta(4\pi G\rho_0 - k^2 c_s^2)$
- K- and E-corrections: $m_{BP} = M_{BP} + 5\log(10{\rm pc}/{\rm d_L}) + {\rm k_{BP}(z)} + {\rm e_{BP}(z)}$