

# Calculus in several variables

General exam 15.11.2012

1. Is the function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ ,

$$f(x) = \begin{cases} (x_1^3 + x_2^3 + x_1x_2)/\|x\|, & x \neq 0, \\ 0, & x = 0, \end{cases}$$

differentiable at the origin?

2. Prove that the equation  $x^2 - ze^{x+y+z} = 0$  defines a surface in some neighborhood of the origin. Also, determine the equation of the tangent plane to this surface at the origin.
3. (a) Assume that the function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  has partial derivatives at the origin with respect to both variables. Does it follow that  $f$  is continuous at origin?
- (b) How is defined the derivative of the function  $f$  at  $x$  to the direction of the vector  $\alpha$ ,  $|\alpha| = 1$ ?
4. Determine the maxima and minima of the function  $f$ ,  $f(x, y) = x^3 - xy^2$  in the set  $\{(x, y) \in \mathbb{R}^2; x^2 + y^2 \leq 1\}$ .
5. Let  $D = \{(x, y) \in \mathbb{R}^2; 1/4 \leq x^2 + y^2 \leq 4\}$ . Compute the integral

$$\int_D (x^2 + y^2)^2 \cos(x^2 + y^2) dx dy.$$