

Matematiikan ja tilastotieteen laitos
 SCIENTIFIC COMPUTING/ MATEMATIIKAN MENETELMÄKURSSI
 Exam, 2012-12-10
 VASTAA NELJÄÄN TEHTÄVÄÄN! / ANSWER FOUR PROBLEMS!
 MATLAB-TEHTÄVÄT SAA TEHDÄ MYÖS MUILLA KIELILLÄ!

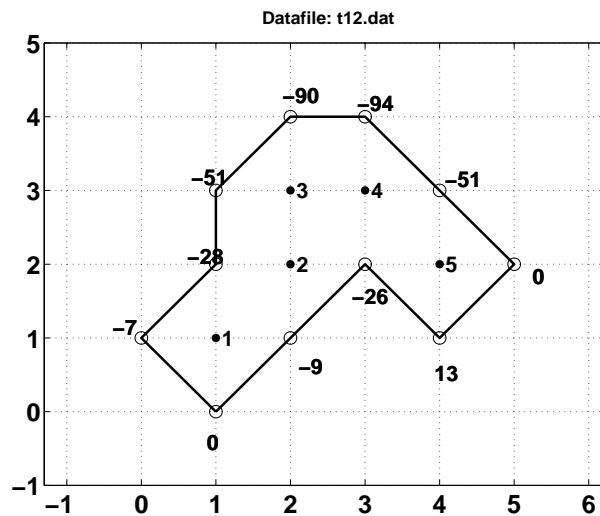
1. Consider the system of equations

$$\begin{cases} x^2 - 2x - y + 0.5 = 0; \\ x^2 + 4y^2 - 4 = 0; \end{cases}$$

- (a) Form its Jacobian matrix J and the inverse matrix J^{-1} . Recall from linear algebra that $[a, b; c, d]^{-1} = T[d, -b; -c, a]$ if $1/T = ad - bc \neq 0$.
- (b) Write Newton's iteration formula for the solution of this system of equations.
- (c) Start the iteration at the point $[2, 0.25]$ and carry out one iteration by hand.
- (d) Write a MATLAB program for the purpose.
2. Solve the Dirichlet problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

in the case of the picture and use the indices 1, ..., 5 to label the unknowns. The side of each square is 1. Carry out all the computations by hand. Also give a MATLAB program for the purpose.



3. Consider the boundary value problem $y'' + \lambda y = 0; y(0) = y(1) = 0$.

(a) Use the approximation

$$y''(x) \approx \frac{y(x+h) + y(x-h) - 2y(x)}{h^2}$$

to formulate the discrete version of this problem at the points $x_k = k/(N+1), k = 1, \dots, N$.

- (b) Carry out the computations by hand when $N = 3$.
 (c) Write a MATLAB program for the case of a general N .

4. Suppose that the functions $\varphi_k : [a, b] \rightarrow \mathbb{R}, k = 1, \dots, n$ are linearly independent functions in $C([a, b])$ and $f \in C([a, b])$ is given. Then the least squares approximation of f in the subspace of $L^2([a, b])$ spanned by $\{\varphi_1, \dots, \varphi_n\}$ is given by

$$f^* = \sum_{j=1}^n c_j^* \varphi_j,$$

where c_j^* satisfy the normal equations, i.e.

$$\sum_{j=1}^n (\varphi_j, \varphi_k) c_j^* = (f, \varphi_k), k = 1, \dots, n.$$

(a) Use this to find the best approximation for $\sin(x)$ in the interval $[-\pi/2, \pi/2]$ in terms of linear combinations of $\{1, x\}$. In other words, find coefficients a, b such that the integral

$$I(a, b) = \int_{-\pi/2}^{\pi/2} (\sin(x) - a - bx)^2 dx$$

attains its least value.

(b) Write a MATLAB program to solve the general problem, when we have at most five functions.

5. Given the data $(x_j, y_j), j = 1, \dots, m, m > 5$, we wish to fit to this data a parabola through the point (x_3, y_3) .

(a) Write the equation of parabola through the point (x_3, y_3) in the form $y = ax^2 + bx + c$ and give formulas for a, b, c . How many free parameters we have in these formulas?

(b) Using the result in part (a), we wish to fit this parabola to the given data using the least squares method. Write the formula for the object function that should be minimized, as a part of the solution of this problem. Explain in your own words what is meant by normal equations. Write these equations for this problem and solve them by hand.

(c) Write a MATLAB program that can compute the numerical values produced in part (b).