Linear algebra and matrices I
University of Helsinki, Department of mathematics and statistics
Course examination 17.10.2012

You are allowed to use a calculator but not a book of formulas in the exam.

1. (6 points) The matrix of a system of linear equations has been modified with elementary row operations, and the result is the following matrix. How many solutions does the system of linear equations have? Determine the solutions.

$$\text{a)}\quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad\qquad \text{b)}\quad \left[\begin{array}{cccc|c} 1 & 3 & 0 & 1 & 3 \\ 0 & 0 & 1 & -4 & -2 \\ 0 & 0 & 0 & 0 & 2 \end{array}\right]$$

2. (12 points) Denote $\bar{w}_1 = (1, 2, 0)$, $\bar{w}_2 = (1, 1, -1)$ and $\bar{w}_3 = (1, 4, 2)$. We want to find out if the vectors $\bar{w}_1, \bar{w}_2, \bar{w}_3$ are linearly independent. What kind of an equation should we consider? What is the system of linear equations that the equation corresponds to?

   When the matrix of this system of linear equations is modified with elementary row operations, the following matrix is obtained

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

   What can you say about the linear independence of the vectors $\bar{w}_1, \bar{w}_2, \bar{w}_3$ on the basis of this matrix? Justify your answer.

3. (10 points) Assume that $k \in \mathbb{R}$. What is the determinant of the matrix $A = \begin{bmatrix} k & 1 & 0 \\ k^2 & 2 & 0 \\ 0 & k & k \end{bmatrix}$?

   Determine the values of k for wich A is invertible.

4. (10 points) Denote $\bar{w} = (-2, 1, 1)$, $\bar{v}_1 = (1, 2, 0)$ ja $\bar{v}_2 = (0, 1, -1)$.

   a) Show that the vector $\bar{w}$ is perpendicular to the vectors $\bar{v}_1$ and $\bar{v}_2$.

   b) Show that $\bar{w}$ is perpendicular to every vector in the subspace $\text{span}(\bar{v}_1, \bar{v}_2)$.

5. (10 points)

   a) How is an invertible matrix defined?

   b) Assume that A, B ja C are invertible $n \times n$ matrices ($n \in \{1, 2, \dots\}$). Show that also the matrix ABC is invertible. What is the inverse of ABC?

You can give course feedback via a link that is sent to your email after the exam. However, the questions will be in Finnish. Giving feedback gives you the same amount of extra points as three tasks in a weekly problem sheet.