

Remark. A candidate is allowed to use an short abstract of size A4.

1. Solve the differential equation

$$y' = \frac{1 - x^2}{y^2}.$$

2. (a) Give a differential equation modeling a change in temperature of an object such that the cooling rate of object is proportional to the difference between the temperatures of object and its environment. Additionally, the temperature of environment is supposed to be a constant.

(b) Apply your equation to the following problem: A thermometer, showing 21° inside in a room, is carried out off the building; the outdoor temperature is 9° . After ten minutes, the thermometer shows 12° . When does it show 10° ?

3. Solve the differential equation

$$y'' - 2y' + 3y = \sin x.$$

4. Solve (implicitly) the differential equation

$$\frac{1}{x} + 4x^2 \sin y + (x^3 \cos y)y' = 0.$$

5. Solve the initial value problem

$$y' + y = x\sqrt{y}, \quad y(0) = 4.$$